



Coefficient of Performance Stabilisation in Ground Source Heat Pump Systems

***Jerzy Wołoszyn*^{*1}, *Andrzej Gołaś*²**

¹AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics,
30 Mickiewicza Av., 30-059, Krakow, Poland

e-mail: jwołoszy@agh.edu.pl

²AGH University of Science and Technology, Faculty of Mechanical Engineering and Robotics,
30 Mickiewicza Av., 30-059, Krakow, Poland

e-mail: ghgolas@cyf-kr.edu.pl

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ABSTRACT

The number of installations with ground source heat pumps is steadily increasing. As they involve high investment costs, they require deliberate action and analysis. Research on the influence of design, materials and operating parameters on their coefficient of performance becomes of great importance. In this article the authors propose a new ground source heat pump system with horizontal ground heat exchanger and subsurface irrigation system. In order to examine the possibility of applying the system, the influence of soil moisture content on the heat pump coefficient of performance was investigated in this research. Conducting research on the real object is extremely expensive, so it was decided to conduct simulation studies using the finite element method. The presented results of research confirm that the soil moisture content has the greatest impact on the heat pump system coefficient of performance. The developed ground source heat pump system with subsurface irrigation system allow to reduce the length of ground heat exchanger loop.

KEYWORDS

Ground source heat pump system, Coefficient of performance, Soil moisture content, Subsurface irrigation system, Quasi3D modelling.

INTRODUCTION

Growing demand of energy requires alternative energy sources. Among the equipment for effective energy conversion are heat pumps. For its operation the heat pump requires a heat source, for example: air, water or soil. The heat is transferred from the heat source via a heat exchanger, which in the present case is placed in the ground. The whole system is called Ground Source Heat Pump (GSHP). The different types of GSHPs are open loop and closed loop. The heat exchanger could be vertical or horizontal, however the most widely used are vertical heat exchangers with closed loop [1]. The vertical heat exchangers are expensive although they are very efficient and drilling costs vary depending on the lithological profile of the rock mass. Hence a

* Corresponding author

compromise is made between higher efficiency and cost in horizontal ground heat exchangers which require a larger ground area. In horizontal heat exchangers embedded in soil, long pipes are installed under freezing zone at a depth of 1 to 2 metres through which heat transfer fluid flows and exchanges heat to or from soil [2]. Broad application of this technology has contributed to recent research concerning heat pump with vertical and horizontal heat exchanger. For example, Benli [3] made a performance comparison between horizontal and vertical source heat pump systems. A detailed literature review with possible applications and challenges of optimal control was made in Soni *et al.* [4] and Atam and Helsen [5]. Babak *et al.* [6] considered the possibility of heat pump use in hot water supply systems. Sivasakthivel *et al.* [7] optimized eight important parameters of Ground Heat Exchanger (GHE). The presented results showed a 15.17% reduction in the length of ground heat exchanger, 2.5% increase in Coefficient of Performance (COP) and 17.1% reduction in thermal resistance of GHE. Hanuszkiewicz-Drapała and Składzień in [8], a mathematical model for Vapour-Compressor Heat Pump (VCHP) ground heat exchanger systems was briefly presented and the cumulative energy and ecological results of the application of VCHP-GHE systems were estimated. Kupiec *et al.* [9], presented the mathematical model of a horizontal ground heat exchanger. Luo *et al.* [10], presented ground investigations for different types of GSHP systems. Based on recently published literature [11], a simple heat and moisture transfer model to predict ground temperature for shallow ground heat exchangers was considered. The model includes the influence of water content on soil thermal properties. Ji *et al.* in [12], an improved 3-D line source model was developed. It allowed calculating the soil temperature distribution and inlet/outlet fluid temperature. Shang *et al.* [13] presented an unsaturated three-dimensional model to study a ground source heat pump system based on moisture porous theory and soil moisture dynamics. Based on literature search, it could be concluded that modelling is an important area for study of vertical or horizontal source heat pump systems. It happens because conducting research on the real objects could be extremely expensive.

There are a number of parameters which determine the efficiency of a horizontal heat exchanger and they can be classified as:

- Basic design and material parameters;
 - Diameter of the heat exchanger pipe;
 - Wall thickness of the pipe and the associated thermal resistance;
 - Distance between pipes axes;
 - Thermal conductivities of the soil and the pipe;
 - Moisture content of the soil;
 - Heat exchanger length;
 - Arrangement in the subsurface;
 - Infiltration and ground water flow;
- Operating parameters;
 - Type of flow (turbulent or laminar);
 - Flow fluid rate;
 - Temperature of the fluid;
 - Distance from the heat consumers;
 - Type of work;
 - Local climatic conditions.

So far, studies allow only a general statement that the thermal conductivity of the soil increases with increasing moisture content of the soil determined by volume or by mass [14, 15]. However, there is no research to indicate the soil moisture at which a specified COP is obtained. For this reason, the characteristic feature of the Heat Pump (HP) with horizontal GHE is variable COP, which is dependent on weather conditions.

After literature review, it became clear that there is a need to conduct sensitivity analysis of GSHP performance on soil moisture content. Most of the research has focused only on the geometrical parameters. For example, Nam and Chae [16] conducted research at different conditions of design and installation with variables such as pipe spacing, installation depth, pipe diameter, circulation water temperature, flow rate, and operation condition. In work presented by Congedo *et al.* [17], a comparative analysis of three types of horizontal ground heat exchangers, to be coupled with water to water heat pumps, has been performed. Further, Simms *et al.* [18] examined the annual performance of horizontal ground heat exchanger in soils with heterogeneous thermal conductivity. In work by Wołoszyn and Gołaś [19], the authors conducted the sensitivity analysis of efficiency of thermal energy storage on selected rock mass and grout parameters using the design of experiments method.

In the present paper, the authors propose a new GSHP system with horizontal ground heat exchanger and subsurface irrigation system (Figure 1). In the proposed system, the method of stabilization of the HP COP with horizontal ground heat exchanger (7) is as follows. Soil moisture sensors (3) measure the volumetric water content of soil layer (8) between the horizontal ground heat exchanger (7), and the irrigation system (5). When the volumetric water content decreases below 15%, the control system (1) will run the irrigation system (5). The temperature sensors (4) are located near irrigation system (5) to measure soil temperature above GHE (7) and turn off the installation in case the temperature decreases below 0 °C.

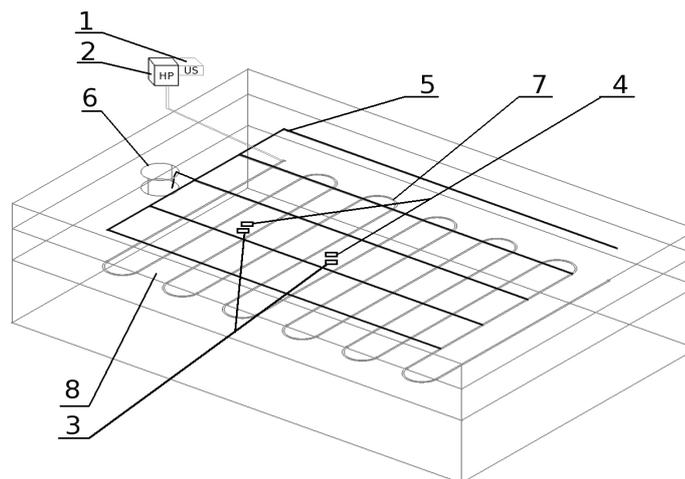


Figure 1. New GSHP system with horizontal ground heat exchanger and subsurface irrigation system: control unit (1), heat pump (2), soil volumetric water content probes (3), temperature sensors (4), irrigation system (5), water reservoir (6), ground heat exchanger (7), soil (8)

In order to verify the proposed solution, the influence of soil moisture content on the HP COP is examined. For the given problem, conducting experiments are very expensive while 3D numerical simulations are time-consuming, therefore the quasi3D heat transfer model was used.

OBJECTIVES OF THE RESEARCH

The objectives of the research are: two configuration of horizontal GHE, which consists of two basic elements: geological medium and heat exchanger, which links energy source with consumer (Figure 2). The calculations were carried out for the soil of the volume of respectively, 7,779 m³ and 8,316 m³. The first configuration consist of 16 GHE loops with a total length $L = 480$ m, and the second consist of 20 GHE loops with a total length $L = 600$ m.

Detailed parameters of horizontal GHS are as below:

- Outer diameter of heat exchanger pipe $d_{p,o} = 40$ mm;
- Thickness of pipe wall $g = 3.7$ mm;
- Length of heat exchanger pipe for the first GHE, $L1 = 480$ m;
- Length of heat exchanger pipe for the second GHE, $L2 = 600$ m;
- Distance between pipe axes, $e = 0.55$ m.

To achieve the aim of this work, the steady state analysis was conducted assuming average annual soil temperature.

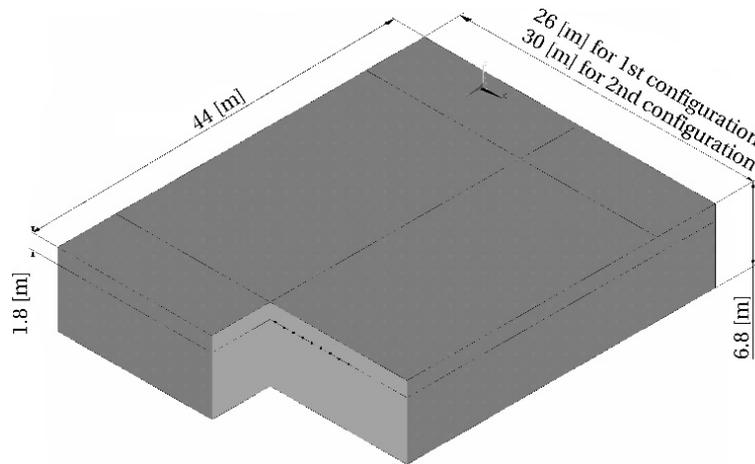


Figure 2. Assumed computational domain

MATHEMATICAL DESCRIPTION

Heat transport in a horizontal ground heat exchanger is described by partial differential equations. Because constant parameters of the system have been considered, steady state conditions were analysed. Therefore, other influential parameters like specific heat and density were not taken into account. Based on literature review [14, 15], it was assumed that the thermal conductivity of the ground was a function of moisture content of the ground. In this case, the heat transport in the ground was derived from Fourier's law and conservation of energy, which present a partial differential equation of steady-state heat conduction eq. (1). Evaporation and seepage of water has been neglected. To describe the heat transport in the fluid and in the pipe the modified quasi3D model [20, 21] was used. Eq. (2) presents a one-dimensional convection-conduction model:

$$\frac{\partial}{\partial x} \left(\lambda_s(w) \frac{\partial T_s(x, y, z)}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda_s(w) \frac{\partial T_s(x, y, z)}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda_s(w) \frac{\partial T_s(x, y, z)}{\partial z} \right) + b_{fs}(T_f(z) - T_s(z)|_{fs,wall}) = 0 \quad (1)$$

$$\rho_f c_f u_f \frac{\partial T_f(z)}{\partial z} - \frac{\partial}{\partial z} \left(\lambda_f \frac{\partial T_f(z)}{\partial z} \right) - b_{fs}(T_s(z) - T_f(z)) = 0 \quad (2)$$

In order to determine the heat transfer coefficient b_{fs} between fluid and soil, the thermal resistance was calculated. Thermal resistance is defined on the basis of the material and geometrical parameters of the exchanger. Thermal resistance between a fluid and soil is defined by convection of the heat carrier within the pipe element and heat conduction through the pipe wall. It is a classical example of heat transport through a cylindrical wall. The heat transfer coefficient b_{fs} is calculated according eq. (3):

$$b_{fs} = \frac{1}{R_{fs}L_{fs}} \tag{3}$$

where thermal resistance between fluid and soil is defined by eq. (4) and L_{fs} is the circumference of the pipe:

$$R_{fs} = R_{conv} + R_{cond} \tag{4}$$

The thermal resistance due to convection R_{conv} in the pipe element is described by eq. (5):

$$R_{conv} = \frac{1}{\alpha 2\pi r_{p,i}} \tag{5}$$

where:

$$\alpha = \frac{Nu\lambda_f}{d_{p,i}} \tag{6}$$

Because the Nusselt number Nu depends on the type of flow, it was defined in accordance with [22] and modified Gnielinski correlation in the following way as per eq. (7):

$$Nu = \begin{cases} \frac{4.364 \text{ for } Re < 2,300}{\left(\frac{\xi}{8}\right) \times Re \times Pr} \left[1 + \left(\frac{d_{p,i}}{l_p}\right)^{\frac{2}{3}} \right]} & \text{for } Re > 10^4 \\ (1 - \gamma) \times 4.364 + \gamma \left\{ \frac{\left(\frac{0.0308}{8}\right) \times 10^4 \times Pr}{1 + 12.7 \sqrt{\frac{0.0308}{8}} \times (Pr^{\frac{2}{3}} - 1)} \left[1 + \left(\frac{d_{p,i}}{l_p}\right)^{\frac{2}{3}} \right] \right\} & \text{for } 2,300 \leq Re \leq 10^4 \end{cases} \tag{7}$$

where Pr represents the Prandtl number and Re , the Reynolds number:

$$Pr = \frac{c_f \mu_f}{\lambda_f}, \quad Re = \frac{\rho_f u_f d_{p,i}}{\mu_f}$$

$$\xi = (1.8 \log_{10} Re - 1.5)^{-2}$$

$$\gamma = \frac{Re - 2,300}{10^4 - 2,300} \text{ and } 0 \leq \gamma \leq 1$$

Thermal resistance as a result of conduction within the pipe material is determined by the following equation:

$$R_{cond} = \frac{\ln\left(\frac{r_{p,o}}{r_{p,i}}\right)}{2\pi\lambda_p} \tag{8}$$

Boundary conditions

Solving eqs. (1) and (2) requires assuming the boundary conditions. It was assumed that the soil temperature increases with depth according to an average annual soil

temperature in Krakow, Poland (Figure 3). To simulate external weather conditions, for the top surface of the research model the convection boundary condition was assumed as in eq. (9):

$$-\lambda_s \frac{\partial T_s}{\partial n} = b_{as}(T_s - T_a) \tag{9}$$

The heat transfer coefficient b_{as} between air and soil surface equal $b_{as} = 7 \text{ W/mK}$ with air temperature equal $T_a = 280.15 \text{ K}$. For the lateral surface, the Dirichlet boundary condition was assumed and temperature varied with depth in accordance with Figure 3.

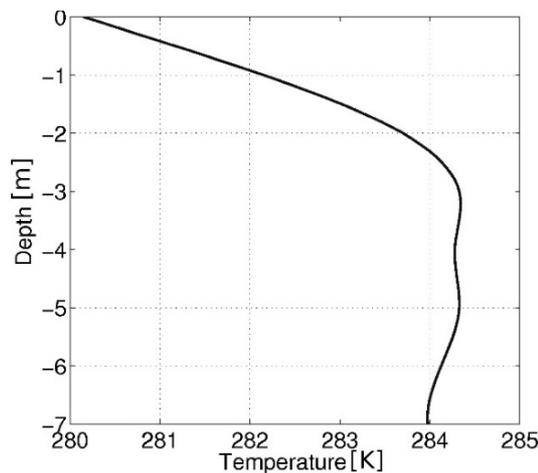


Figure 3. Temperature changed with depth – boundary conditions

As an inlet boundary condition, an annual average inlet temperature of the fluid in the horizontal heat exchanger was assumed $T_f = 273.15 \text{ K}$. The fluid velocity $u_f = 1 \text{ m/s}$. Soil thermal conductivity changed with volumetric water content and take values according measurements conduct in [15] and presented in Figure 4 and Table 1. One of the most used fluids in GSHP is a water and glycol mixture. The assumed values of specific thermophysical constants of propylene glycol are as follows: dynamic viscosity $\mu_f = 0.0052 \text{ Pa s}$, density $\rho_f = 1,052 \text{ kg/m}^3$, thermal conductivity $\lambda_f = 0.48 \text{ W/mK}$ and specific heat $c_f = 3,795 \text{ J/kgK}$.

To ensure that the results would converge, a grid independence test was made (Figure 5). The numerical simulation model contains 1,190,280 cells.

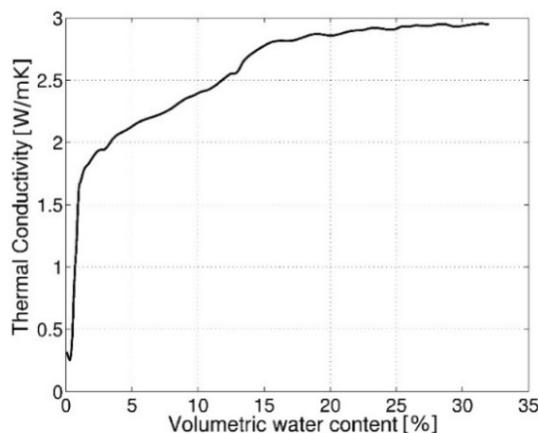


Figure 4. Thermal conductivity in relation to volumetric water content for loosely packed sand [15]

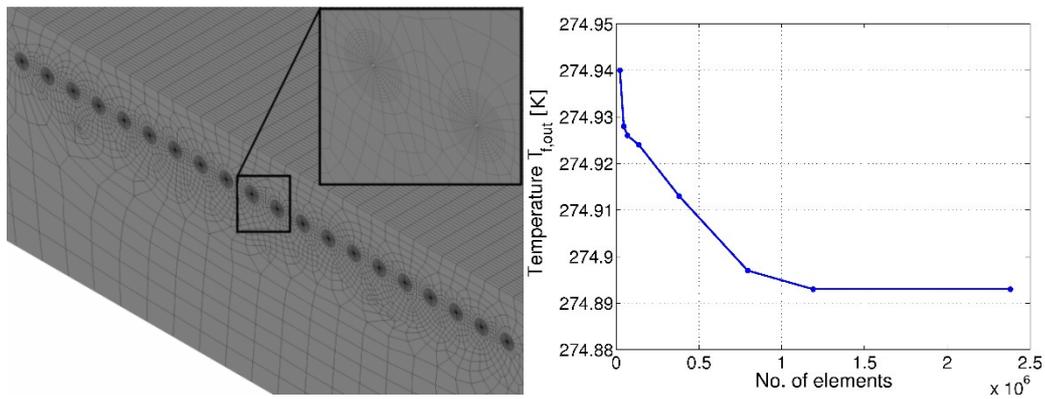


Figure 5. Grid and grid independence test

RESULTS OF NUMERICAL CALCULATIONS

To realize the aim of this research, the quasi3D numerical model of heat transport was used. The sparse direct solver was used to solve the partial differential equations described by the model. As output parameters the coefficient of heat pump performance was calculated according well known equation as the ratio of the heat removed from the cold reservoir plus the input work to the input work and in the flow system:

$$COP = \frac{\dot{Q}_c + \dot{W}}{\dot{W}} \quad (10)$$

where the removed heat flow from cold reservoir is:

$$\dot{Q}_c = \dot{m} \times c_f \times \Delta T_f \quad (11)$$

and mechanical power consumed by the compressor is $\dot{W} = 2$ kW, e.g., small domestic heat pump system. In the analysed issue it was assumed that the compressor works at constant power.

In Table 1 the variation of soil volumetric water content, thermal conductivity and COP results for the first and second configuration, are shown. The influence of moisture soil content on the heat pump COP was examined. The sensitivity was calculated as derivative COP and soil volumetric water content according to eq. 12:

$$COP_{Sensitivity} = \frac{dCOP}{dw} \quad (12)$$

Table 1. COP, soil moisture content and thermal conductivity

Volumetric water content [%]	Thermal conductivity [W/(mK)]	COP 1 st config. [-]	COP 2 nd config. [-]
0.07	0.2	1.3	1.3
0.3	0.4	1.5	1.6
0.6	0.6	1.7	1.9
0.62	0.8	1.9	2.15
0.77	1.1	2.26	2.53
0.89	1.4	2.55	2.87
1.7	1.8	2.9	3.29
6.5	2.2	3.2	3.66
12.2	2.5	3.43	3.91
15.55	2.8	3.64	4.16

Figure 6 presents the obtained local sensitivity. As can be seen, the soil moisture content has a significant impact on COP, especially to 1% of soil volumetric water content. This is because with the increase in soil moisture content, the thermal conductivity increases rapidly.

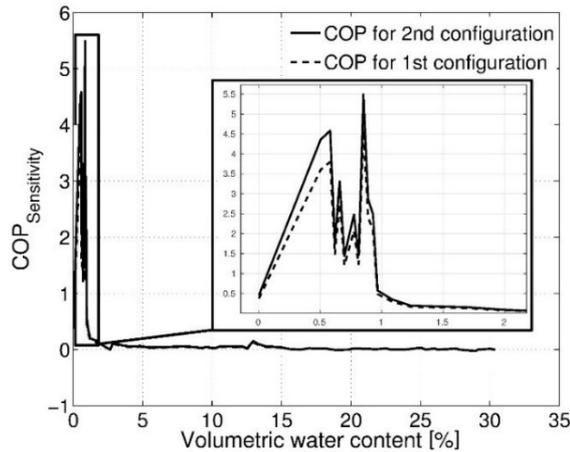
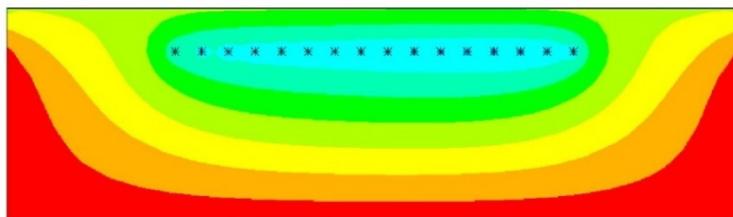


Figure 6. Local sensitivity chart

Figure 7 presents temperature distribution in the ground for thermal conductivity of 0.2 W/(mK) and 2.8 W/(mK). Figures 8 and 9 present COP in relation to thermal conductivity and in relation to soil volumetric water content, respectively. On Figure 8 it can be seen that with the increase of thermal conductivity, the COP increases. It is due to an increase in heat source efficiency. The same occurs in the second heat exchanger configuration ($L = 600$ m). Presented results confirm the correctness of the calculation. In Figure 9 it can be seen, that in the volumetric water content range of 0-1% there is a rapid change of COP 1.5-2.8 for the first configuration and 1.5-3.2 for the second configuration. However, for volumetric water content range of 1-30%, the COP changes in the range of 2.8-3.7 and 3.2-4.25, respectively.

Temperature [K]

for $\lambda_s = 2.8$ W/(mK)



for $\lambda_s = 0.2$ W/(mK)

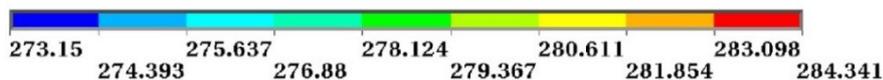
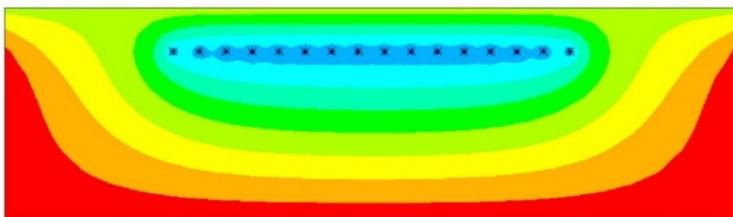


Figure 7. Temperature distribution in the ground for 1st configuration

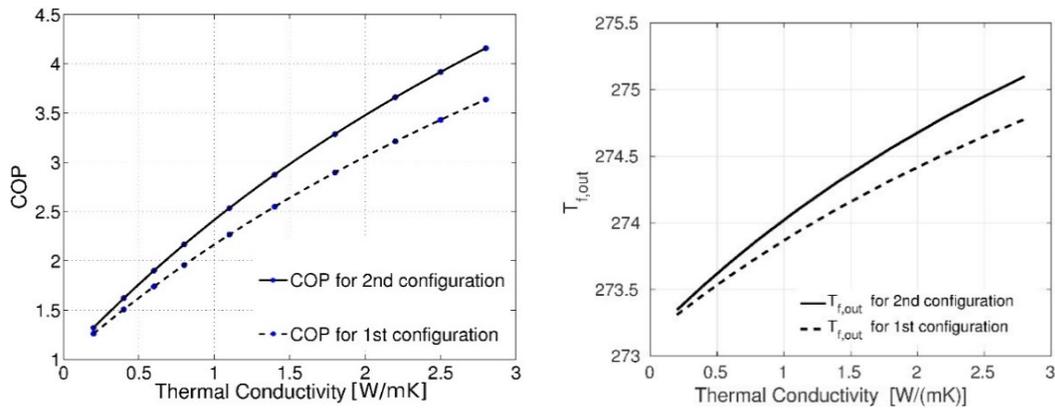


Figure 8. COP and fluid output temperature in relation to thermal conductivity

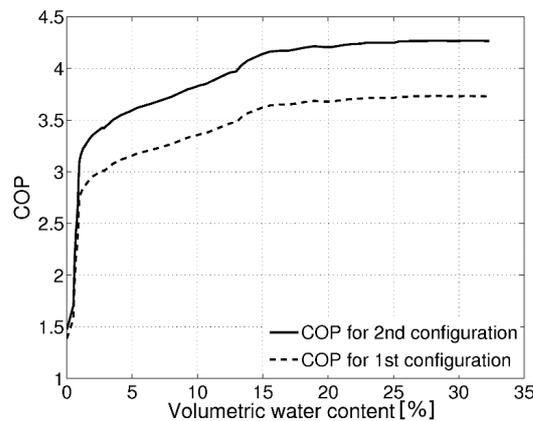


Figure 9. COP in relation to soil volumetric water content

CONCLUSION

In this work a new numerical heat transport model in horizontal heat exchanger was described. Work begins from the theory concerning the construction of a one dimension fluid-thermal finite element. In order to obtain a more accurate model, the soil was divide for 3D element. The calculations were performed using an Intel Quad 2.33 GHz processor. The soil moisture content has a significant impact on COP, especially to 1% of soil volumetric water content. This is because increasing soil moisture content rapidly grow the thermal conductivity. As soil, a loosely packed sand was assumed. Proposed new GSHP with irrigation system as way to regulate soil moisture content reduces the amount of used GHE loop according to the required value of HP COP. If the volumetric water content is constant and equal about 14% the GHE loop will be shorter about 20% to required COP = 3.5. It should be borne in mind that the negative external air and soil temperature will be reduced energy efficiency of the installation due to the lack of soil irrigation.

NOMENCLATURE

b	heat transfer coefficient	[W/(m ² K)]
c	specific heat	[J/(kgK)]
d	diameter	[m]
e	distance between pipes axes	[m]
g	thickness of pipe wall	[m]
L, l	length	[m]
\dot{m}	mass flow	[kg/s]

\dot{Q}	heat flow	[W]
R	thermal resistance	[(mK)/W]
r	radius	[m]
S	sensitivity	[-]
T	temperature	[K]
u	velocity	[m/s]
w	soil volumetric water content	[%]
\dot{W}	power	[W]
x, y, z	coordinates	[m]

Greek letters

α	heat transfer coefficient	[W/(m ² K)]
λ	thermal conductivity	[W/(mK)]
ρ	density	[kg/m ³]
μ	dynamic viscosity	[Pa s]

Subscript

a	air
c	cold reservoir
cond	conduction
conv	convection
d	heat source
f	fluid
g	heat sink
i	in, inner
o	out, outer
p	pipe
s	soil

Abbreviations

COP	Coefficient of Performance
GHE	Ground Heat Exchanger
GSHP	Ground Source Heat Pump
HP	Heat Pump
VCHP	Vapour-Compressor Heat Pump

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