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Modelling of smart irrigation with replan and redistribution algorithms

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ABSTRACT

Climate change is strongly affecting the availability of water. The demands of water in agriculture are by far the largest, where70% of the water used in Portugal, is used in agriculture, and, from that 40% are lost due to inefficiency**.** To develop intelligent irrigation systems to save water is a priority. Current irrigation systems mostly have a controller of the ON-OFF type. The irrigation with ON-OFF controllers might experience limitations, especially when the crop water needs are small and sampling periods are large. Using optimal control formulations and techniques, the water consumption can be made to follow more closely the hydrological needs of the crop, while also taking into account current weather conditions. In this paper, the mathematical model presented by the authors in previous publications is improved. This new model incorporates new features like the slope of the soil, the possibility to include a percentage of water losses due to runoff, and a percentage of water losses if the soil is on the field capacity. A new and more efficient replan strategy is applied tacking into account the data measured from moisture sensors, to ensure that hydric needs of the crop is fulfilled. A new approach to deal with multiple irrigation points is also proposed. It allows to redistribute the available water in the case an irrigation point is not able to provide the water needed, accordingly to its irrigation plan.

KEYWORDS

Smart irrigation, irrigation plan, replan, water redistribution algorithm, soil moisture.

INTRODUCTION

Nowadays climate changes are occurring at a fast pace. According to [1], climate change is estimated to have a significant impact on the water cycle, affecting rainfall patterns and altering the availability as well as quality of both underground and surface water, agricultural production and associated ecosystems. It urges that human attitudes change drastically to revert this situation. It is known [2] that, if nothing is done to overcome this issue, the global warming will increase. Temperature will raise, longer and more frequent drought periods, as well as other extreme weather conditions, will occur. Many regions of the world are facing severe water scarcity. Irrigation becomes an important issue. In [3] the authors present a review on

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the global research in sustainable irrigation in agriculture over the last two decades and the main agents promoting it, the topics that have received the most attention. The main concerns are the improvement of irrigation water use, to have a more efficient agriculture (producing more using less water resources) and mitigate the effects of global climate change. The authors also state that research on sustainable irrigation agriculture should be made to make this technology accessible, as its cost is economically unfeasible for small-scale agriculture in many countries.

Nowadays, decision support systems are widely used in the agricultural sector. They have become indispensable tools to conduct a more sustainable and productive agriculture which is difficult to sustain due to the continuous climate changes [4]. In [5], the authors study how the variation of the climate, the climate change, and the siltation of reservoirs, will affect the use of land, and how the population growth will challenge the sustainability of engineered river systems in the near future. In [6], the authors present a study on regional climate change impacts on irrigation vulnerable season shifts in agricultural water availability for South Korea. As the authors note, reservoirs are the main source of irrigation water in South Korea. However, to properly manage reservoirs, it is necessary to know how long are the water shortages.The authors assess the vulnerable seasons of paddy irrigation, enabling a more efficient management of the reservoirs. To do so, the authors evaluate the vulnerable seasons taking into account the potential water supply capacity and irrigation water needs, using time series.

In many countries, such as Portugal, most of the water resources are used in irrigation of crop fields. Hence, it is crucial to develop an efficient irrigation plan that is able to maximize the production of a crop field with the minimum amount of water possible [7]. The report of G20 [1] has as first recommendations to modernize irrigation schemes. It promotes the modernization of large scale surface irrigation systems in order to improve irrigation efficiency. The study of irrigation systems and related infrastructure has been addressed previously using Genetic Algorithms. In [8] a model based on on-farm irrigation scheduling and the simple genetic algorithm optimization (GA) method for decision support in irrigation project planning. In [9], a multi-objective fuzzy stochastic model for determination of optimum cropping patterns with water balance for the next crop season was developed. In [10], a GA technique is used to evolve efficient cropping pattern for maximizing benefits for an irrigation project in India. In [11], the authors present an automatic recommendation irrigation system for the purpose of minimizing the water use, where big data analytic techniques have been applied to predict the optimal irrigation. In [12], the authors present a model to calculate the soil moisture when data is scarce. They use soil moisture measurements every two weeks, and use a Random Forests algorithm to estimate daily soil moisture, converting the original data, to data with a daily time step. This is very important to any irrigation system where data on soil moisture is scarce. In [13], an optimization model for the allocation of agricultural irrigation water to improve irrigation water use efficiency considering uncertainties of fluctuating water supply is presented. To demonstrate its feasibility, the proposed models are applied to an irrigation district in northeastern China. In [14], six irrigation treatments were applied combining different levels of water deficit depending on the phenological stage, the models AquaCrop and MOPECO were compared with simulations. Other authors like the ones of [15] present a review on deficit irrigation (DI) management practices (reducing the amount of water provided to the crop during the growing season to a level below that needed for maximum plant growth) able to minimize the impact on crop yield. They also remark that attention should also be towards studying other crops that could withstand deficit irrigation with minimum impact on yield and quality.

Optimal control theory emerged in the 50s to solve problems concerning space [16] exploration of the solar system [17]. It is a recognized tool used in areas such as robotics , biological systems [18], health problems [19], economy problems [20], among many others. The goal of optimal control theory is to find a control law for a given system such that a certain optimality criterion is achieved.

This paper proposes to study irrigation systems as an optimal control problem. Previous works by the same authors have shown that when an irrigation system is modelled as an optimal control problem, the total amount of water spent can be reduced. One may read this in [21] and [22]. In [21] and [22] the solution for a similar problem was numerically validated, taking into account theoretical developments in Optimal Control, namely the Maximum Principle [23]. The analytic solution was obtained and it was compared with the numerical one. In order to understand the gains of use this approach a comparison with real case was presented.

This paper proposes and investigates an improved formulation of an optimal control model, enhancing results previously reported in [21] for obtaining a plan of irrigation of a crop field for a given period of time.

The optimal control model in [21] was formulated to minimize the water used while keeping the crop safe, where the Horton's equation [24] and Soil water balance model [25] were considered to estimate losses of water due to deep infiltration. New features like a slope of the soil, the possibility to include a percentage of water losses due to runoff, and a percentage of water losses if the soil is field capacity are added to previous models by the authors.

Moisture sensors in the crop field are used to measure the real trajectory of the state variable. Considering that several unforeseen events can occur, such as poor performance of the irrigations system, climatic changes that were not foreseen, it is necessary to check if the predicted soil moisture corresponds to the actual humidity. If the correspondence is not verified, corrections must be made by replanning. In [21] a replan strategy was presented have in consideration the data of the precipitation, however, it was formulated via penalizing the cost function. However it may be difficult to use because the constant of the penalization is not known a priori. To overcome this issue, this work proposes a new reformulation of the optimal control problem. In the new proposed problem, instead of penalizing the objective function, a new constraint is incorporated to impose that the soil moisture does not violate the state constraint (soil moisture is set with minimum value to guarantee the crop is safe), which will force to add the necessary value to the control variable (the amount of water to irrigate). To do it, in this work a new replan algorithm is also proposed, for obtaining a new irrigation plan based on real updated values of the soil moisture trajectory.

A new approach to deal with several irrigation points is also proposed. It allows to irrigate rectangular crop fields that may have different crops, different types of soil, and different slopes. It also allows to redistribute the available water in the case an irrigation point is not able to provide all that is needed accordingly to its irrigation plan. This is an important step towards obtaining a more realistic approach. A qualitative validation is performed via the creation of various scenarios.

This paper is organized as follows. First the optimal control model of a "smart" irrigation is presented. When weather predictions are incorrect this means that the moisture of the soil deviates from the predicted during the irrigation plan. To deal with this issue a replan strategy is used for correcting the data, and obtaining a new irrigation plan. The replan strategy is described in following section. Next, the authors discuss a possible way to improve the mathematical model by the use of multiple irrigation points and water redistribution algorithm. After that, the authors present the results obtained by presenting a set of scenarios to validate the mathematical model and also to validate the performance of the replan and water redistribution algorithms. Finally, the conclusions and future work are presented.

MATHEMATICAL MODEL

This section provides a model for a smart irrigation system formulated as the optimal control problem. To solve the optimal control problem, it is discretized and transcribed into a mathematical programming problem, using a thin time-mesh. The objective function to be minimized is the amount of water used in the irrigation system, the sum of the control variables u, subject to M equality constraints and M inequality constraints. The equality constraints imposes that the trajectory of the state variable x (here representing the moisture of the soil) have to satisfy a water balance law. The inequality constraints represent, the satisfaction of the hydric needs (x_{min}) of the crop and the restriction that the amount of water inflow cannot be negative. The proposed formulation of the problem is as described in equations (1) - (4):

$$\min_{u_i, x_i} h \sum_{i=1}^{M} u_i \tag{1}$$

subject to:

$$\begin{cases} x_{i+1} = x_i + hf(t_i, x_i, u_i, x_{i+1}), \qquad i = 1,..M-1. \\ \qquad\qquad\qquad\qquad x_i \geq x_{min} \\ u_i \geq 0, \;\; i = 1,..M \\ x_1 = x_s. \end{cases} \tag{2}$$

where the function f is defined by:

$$\begin{aligned} f = K_I u_i.(1 - R_F.sin(\alpha)) - K_C evtp_0(t_i) + \\ +(1 - R_F\, sin(\alpha)) rfall(t_i) - loss(x_i, x_{i+1}) \end{aligned} \tag{3}$$

with the term loss given by:

$$loss(x_i, x_{i+1}) = \begin{cases} k\, x_i & if\ x_i \leq x_{FC} \\ (x_{i+1} - x_{FC})C_S & if\ x_i > x_{FC} \end{cases} \tag{4}$$

and M is the number of time intervals (days/hours) of irrigation planning, h is the time step, x_S is the soil moisture measured at an initial time by a moisture sensor, and x_{min} is the crop needs of water in order to survive.

It is worth noting that in order to calculate f, in eq. (3), one needs to calculate 4 parcels. The first one, defines the necessary irrigation (u), being K_I a coefficient related to the type of irrigation, α is the angle of inclination of the soil, and R_F is a coefficient that determines the losses due to runoff. The second parcel, similar to the first one, describes how rainfall is involved in the dynamics. The third parcel is due to the crop and its water needs, where K_C is the coefficient related to the type of crop. $evtp_0$ is the reference evapotranspiration value [26]. The fourth parcel defines the water losses, due to the soil characteristics and its moisture level. In order to calculate$loss$, in eq. (4) we needx_{FC}that is the moisture of soil at the available water capacity and C_S is a saturation coefficient. The C_S coefficient allows us to consider that even though the soil may be saturated, it is able to retain above it a certain amount of the excessive water, until it eventually infiltrates the soil. $k$is a parameter associated with the deep infiltration of water in the soil, that has to be studied beforehand and adjusted properly. Infiltration permeability of clay soil was already studied for instance in [27].

The new features of this mathematical model compared to the one presented in [21] is the possibility to introduce the slope of the crop field (and its consequences – runoff), the percentage of water losses due to the field water saturation (eq. (4) of the mathematical model), and the introduction of the soil moisture via a moisture sensor.

REPLAN

Although the model described in the previous section tries to represent the reality, andits evolution given the current state and current inputs, unforeseen events can always occur, such as change of the weather conditions which may lead to poor performance of the irrigation systems. It is important to evaluate if the results obtained via the irrigation planning approximate the reality. If not, there is the need to replan the solution according to the current real data. The replan is done setting the initial state as the actual real moisture in soil, and take into account the new weather predictions. Although replan could be done on an hourly basis, if the irrigation system is not completely automatic, for instance, if once the plan is generated, the farmer has to activate the irrigation system manually, it is more convenient, that its implementation, is done on a daily basis. The model is essentially the same, only the time step h instead of being 1 hour, it is 1 day. Since the goal is to have an irrigation planning system completely automatic, a replan strategy was developed and incorporated into the irrigation planning system. The replan strategy is presented in the next flow chart in Figure 1. Details presented in Appendix 1.

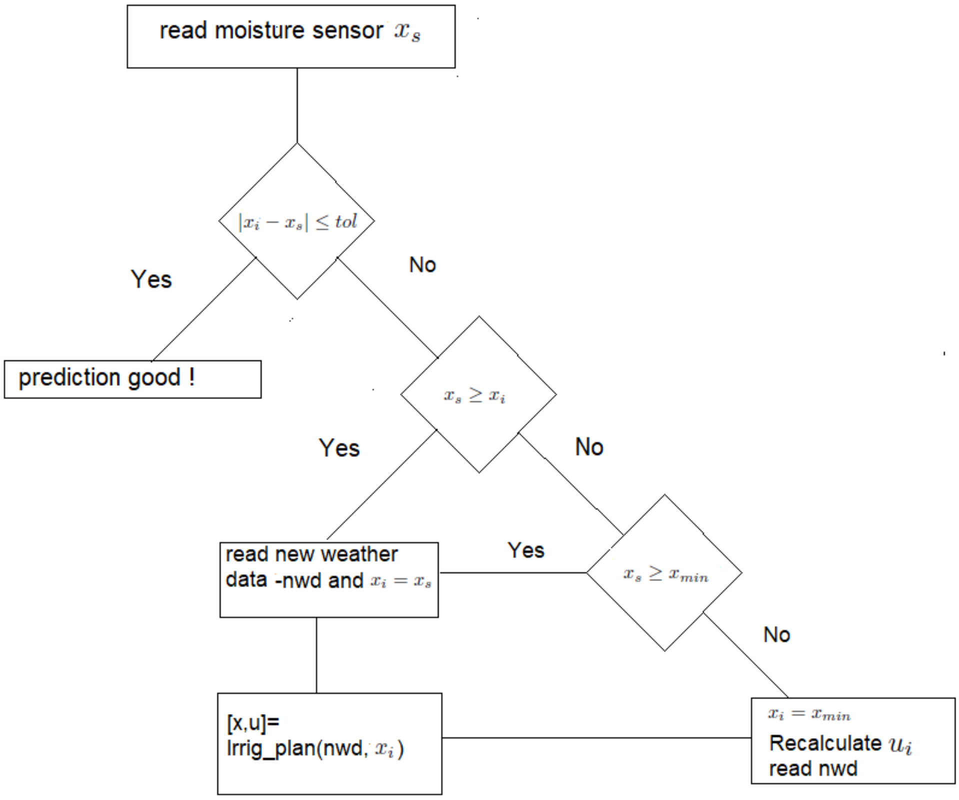

Figure 1: Replan flowchart.

The replan algorithm basically detects if the soil moisture, read by the sensor, is much different than the one the Irrigation plan predicted for the current day. If that is the case, two situations may occur: the state constraint is not violated, and then it basically repeats the process with new data. If the state constraint is violated, the value of irrigation to perform is calculated, such that the state constraint is not violated. New data is considered, imposing that the initial state is nowx_{min}, and the program is rerun. This approach is simpler and different from the one presented in [21], which was based on a penalty approach.

MULTIPLE IRRIGATION POINTS PLAN

So far, the model assumed that the whole field is described as one point in space, where each point represents a unit area. That is, all the measures that place at a certain point in the crop field, and that the irrigation plan for the whole crop field is designed for that point, or the space distribution is uniform. A more realistic model would consider several irrigation points. To simplify the exposition, the model assumes that the field is rectangular, composed by several smaller rectangular areas (example in Figure 2). The model also assumes that at the center of each of the smaller rectangles there exists an irrigation point. For each one of these small rectangles, an irrigation plan as described in the previous sections is obtained. This means that the new irrigation planning system is now able to cope with different needs of irrigation depending on the spatial location.

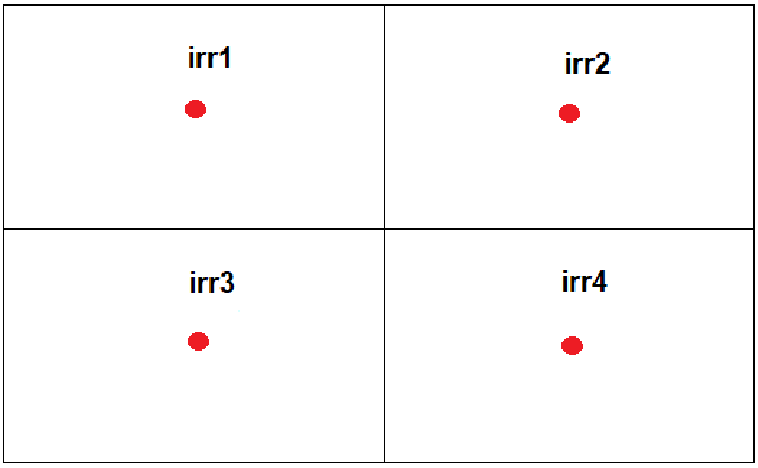

Figure 2: Crop field with multiple irrigation points.

In order to consider multiple irrigation points, the multiple irrigation points plan algorithm (detailed in Appendix 2 – Multiple irrigation points plan algorithm) is proposed and implemented. After obtaining the irrigation plan for each irrigation point, the water redistribution algorithm will verify if each the irrigation point is able to irrigate the necessary amount of water.

In the multiple irrigation points plan algorithm, the following notation is considered: N is the number of irrigation points, ND is number of days the irrigation plan is designed for, P (N; day) is the irrigation plan on certain day (day=1,…,ND) in each of the N irrigation points and C(N) is maximum capacity of each of the irrigation points.

The algorithm verifies if all the irrigation points are able to fulfil the irrigation plan for that day. If, for instance, an irrigation point is not able to fulfil its needs, the water redistribution algorithm seeks the possibility to fulfil the necessary needs in other irrigation points. This information is passed on to the farmer. If it is not possible at all to fulfil the needs of every irrigation point, after fulfilling the needs of irrigation at the points where that is possible, the available water is redistributed in the irrigation points where water is in needed.

Since the information on what are the water deficits (wd) at each irrigation point is passed to the farmer, eventually, he will be able to obtain water elsewhere, and proceed with the multiple irrigation points plan.

RESULTS AND DISCUSSION

In order to validate qualitatively the mathematical model, different scenarios were considered. Although the model assumes only have daily data, an hourly based model will be used (if in the future the data is hourly) allowing us to have a more accurate model.

The data used (weather and local moisture in the soil) to support the findings of this study are available from the corresponding author upon request.

In the following subsections the replan algorithm and the multiple irrigation points plan algorithm will also be qualitatively validated using a set of scenarios.

Mathematical model validation

In order to validate the mathematical model, several scenarios are considered.

Base Scenario 1

For the Base Scenario 1, considers that the weather data, soil data, crop data are read from a text file, and are for a period where rainfall was scarce. The weather data for these days was the following (taken from a weather station placed on the crop field):

Rainfall = [10 0 0 0 0 0 0 10 0 0] (mm)
WindSpeed = [2.8 3 2.4 2.4 3 2.2 2.7 2 2.2 2.8] (m/s)
MinTemp = [17.4 17.1 15.2 15.3 15.1 15.6 16.3 16.3 15.4 14.7] (ºC)
MaxTemp = [24.4 20.8 23.1 22.2 24.9 26.9 29.3 23.6 19.4 20.4] (ºC)

The crop considered was grass, in the region of Oporto (NW of Portugal). The minimum amount of water in the soil is 18.72 mm. In this Base Scenario 1 is also assumed that the field is horizontal and all the excessive water is lost, ($C_S = 1$), if the amount of water in the soil is above the field capacity (the soil is saturated). In order to solve problem (1) taking into account this Base Scenario 1, the problem was written in AMPL [28] and solved by a primal-dual interior-point filter line search method [29], known for IPOPT solver and available in NEOS platform. Local and global convergence proprieties of this method were analyzed in [30]. The optimal solution obtained for irrigation and moisture in the soil is shown in Figure 3.

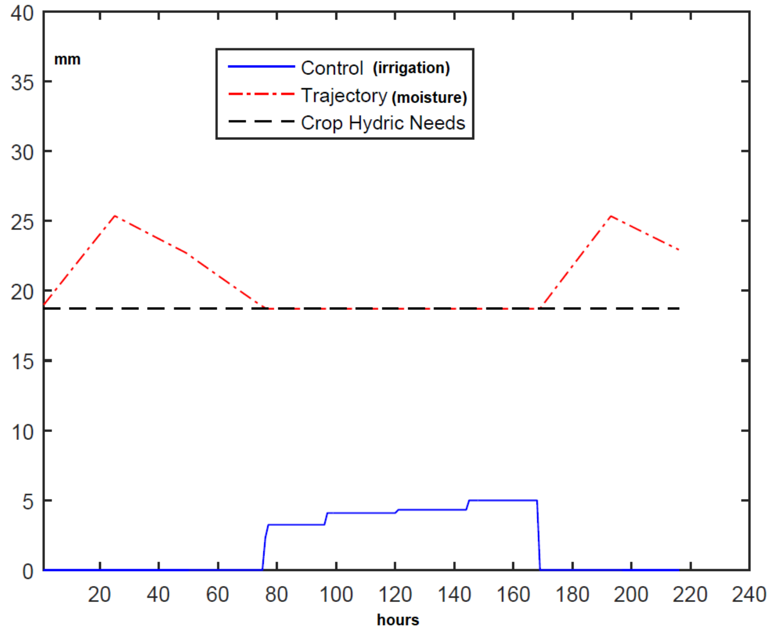

Figure 3 : Base Scenario 1 results when rainfall is scarce.

From Figure 3, it is possible to see that the irrigation is not necessary in the first 3 days and the last 3 days because it rained in the first and eighth day. The total consumption of water was 16.2 mm. The problem was solved in 0.12s of CPU time.

From the Base Scenario 1, the field inclination is changed to $\alpha = 20°$, and it was considered that 50% of runoff is lost (R_F = 0.5), keeping all the other parameter values, it is denominated Scenario 1a. The optimal solution of the problem for the Base Scenario 1a is shown in Figure 4. Note that, there is rain in that period, but due to the field not being flat, runoff losses take place and irrigation is necessary. The water consumption is 23.3 mm. The irrigation starts earlier (see Figure 3). When it rains, soil moisture is lower in comparison with the Base Scenario 1.

Considering the base scenario 1 with the horizontal field again, but with the maximum temperature in the fifth day is 38.9 ºC instead of 24.9 ºC, designated by Scenario 1b. The optimal solution of the problem for the Scenario 1b is shown in Figure 5. The total consumption increases relatively to the Base Scenario 1. As it was very much hotter in one of the days, evapotranspiration of the crop increases a lot, and so the need of irrigation. Therefore, the water consumption increases to 20.3 mm (comparing with Base Scenario 1).

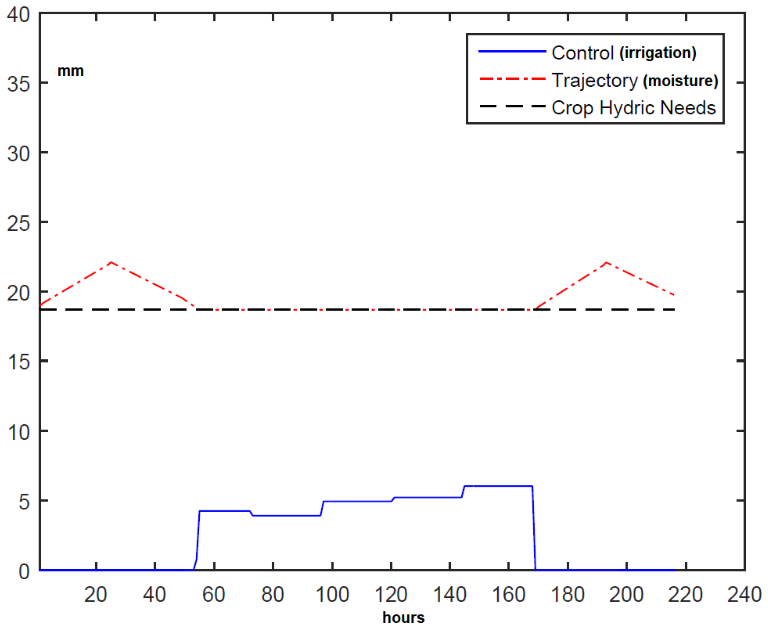

Figure 4: Scenario 1a: α=20º and $R_F = 0.5$.

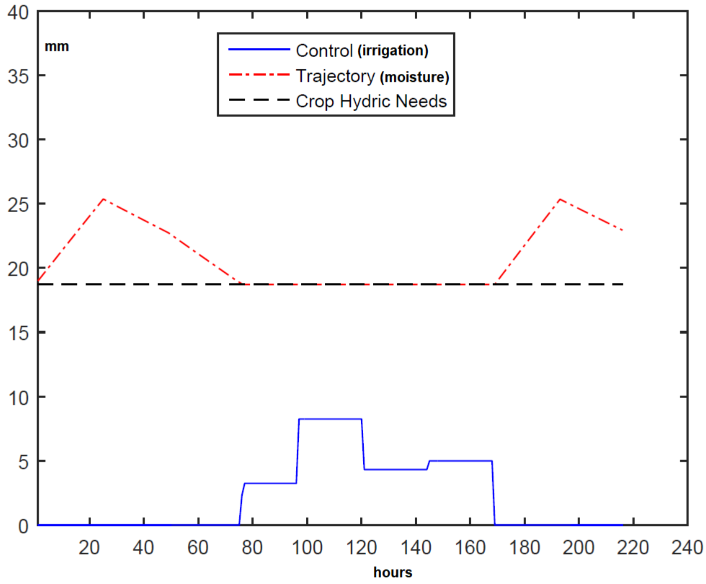

Figure 5: Scenario 1b: the maximum temperature on the fifth day is 38.9 ºC instead of 24.9 ºC.

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Other variations of Base Scenario 1 could be considered. However, from the above scenarios it is possible to conclude that the model behaves well in qualitative terms. The model behaves as expected. It there is a slope in the soil, the runoff is responsible for the increase of the irrigation, and the same happens when the temperature is higher in periods of draught.

Base Scenario 2

For the base scenario 2, the weather data, soil data, crop data are read from a text file, for a regular rainfall period (April period). The weather data for these days was the following (taken from a weather station placed on the crop field):

Rainfall = [5.8 4 13 7.1 4.8 0 0 0 50.3 3.8 3.0] (mm)
WindSpeed = [4.5 7.8 2.9 3.1 2.9 2.7 6.9 2.2 3.2 2.7] (m/s)
MinTemp = [12 14 11 10 9 8 14 15 15 15] (ºC)
MaxTemp = [16.1 17.1 16.4 17.2 16.8 17.1 16.7 16.3 17.2 16.6] (ºC)

Again, the crop considered was grass, in the region of Oporto (NW of Portugal). The minimum amount of water in the soil is 18.72 mm. In the Base Scenario 2 is also considered that the field is horizontal ($\alpha = 0^{\text{º}}$) and all the excessive water is lost, ($C_s = 1$), if there exists saturation. The optimal solution of the problem (1) Based on Scenario 2 is shown in Figure 6. Note that, the irrigation is not necessary because the moisture in the soil is always above x_{min}. Indeed the soil reaches the field capacity at a certain point, $x > x_{FC}$ with $x_{FC} = 46.8$ mm. The total consumption of water was 0mm. IPOPT solver took approximately 0.1 s of CPU time to solve the problem.

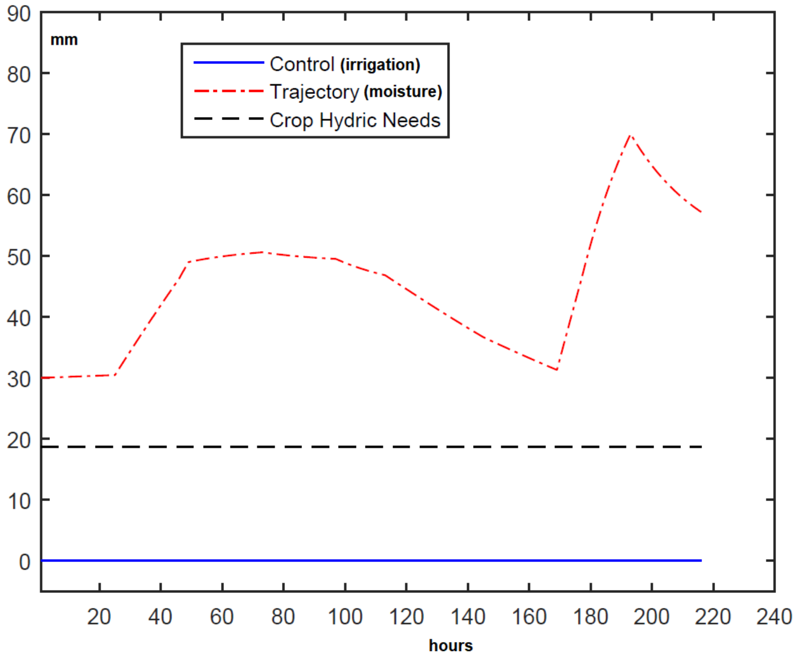

Figure 6: Base Scenario 2 results when rainfall is regular.

In Scenario 2a the weather conditions are the same as in Scenario 2 but its assumed that field has a slope of $\alpha = 20^{\circ}$ and 50% due runoff is lost. The optimal solution of the problem for this scenario is shown in Figure 7.

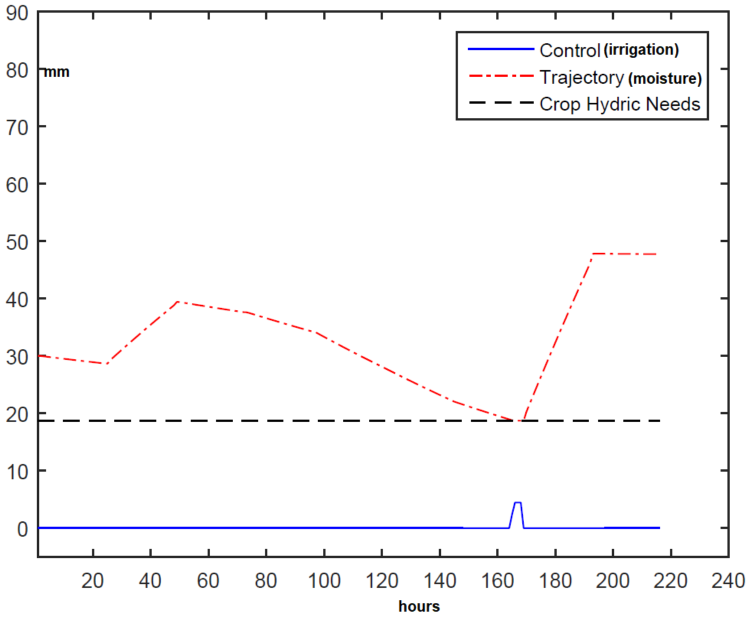

Figure 7: Scenario 2a: $\alpha = 20^{\circ}$ and $R_F = 0.5$.

It is interesting to see (Figure 7) that, if the field is with $\alpha = 20^{\circ}$ inclination and 50% of runoff is lost ($R_F = 0.5$), the amount of water lost by runoff is quite significant. It can lead to a point where, even for a period where plenty of rain fell, it might be necessary to irrigate. This allows us to conclude that, in such a scenario it would be very useful for the farmer to build a tank to collect runoff.

Replan Algorithm Validation

As base scenario to validate the replan algorithm, a very dry period was considered. The weather data for these days are the following:

Rainfall = [0 0 0 0 0 0 0 01.02 0] (mm)
WindSpeed = [2.8 3 2.4 2.4 3 2.2 2.7 2 2.2 2.8] (m/s)
MinTemp = [17.4 17.1 15.2 15.3 15.1 15.6 16.3 16.3 15.4 14.7] (ºC)
MaxTemp = [24.4 20.8 23.1 22.2 24.9 26.9 29.3 23.6 19.4 20.4] (ºC)

It is assumed the measured moisture in the soil (x_s) was close to the one predicted by the initial irrigation plan (trajectory) - no replan was needed. The optimal solution of the problem (1) for the replan base scenario is shown in Figure 8. The total water consumption was 33.35mm. The real moisture in the soil was:

x_s= [19 18.7 18.7 18.7 18.7 18.7 18.7 18.7 18.7 18.7] (mm)

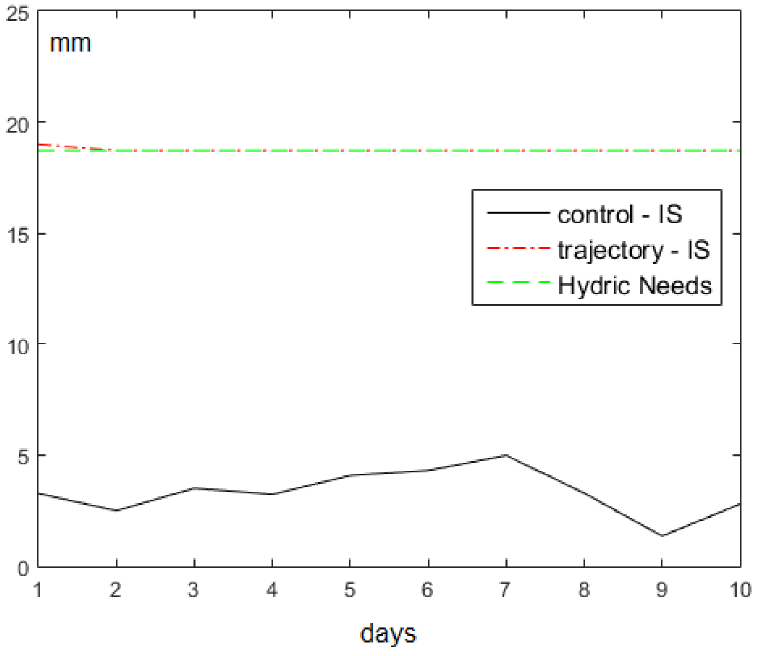

Figure 8: Replan base scenario - set of very dry days, and, real values for the necessary data are as expected.

Let us now assume that the moisture sensor measures a different value(lower) than expected on the fifth day:

x_s= [19 18.7 18.7 18.7 15 18.7 18.7 18.7 18.7 18.7] (mm)

Replan will be activated. The optimal solution of the problem (1) for this scenario can be seen in Figure 9. The hydric needs are the needs of the crop. It is important to satisfy this constraint. When this constraint is violated ($x_s < x_{min}$), it is imperative to correct the needs of the water to avoid the crop to suffer/die - second part of the replan algorithm described before.

As it can be seen in Figure 9, in the first four days the initial planning solution (solution IS: control IS and trajectory IS) is equal to the solution obtained in the replanning (solution R: Control R and Trajectory R). After that, the needs of water exceed the planning solution at the fifth day. In this scenario, the amount of water used by the irrigation systems was 36.4 mm.

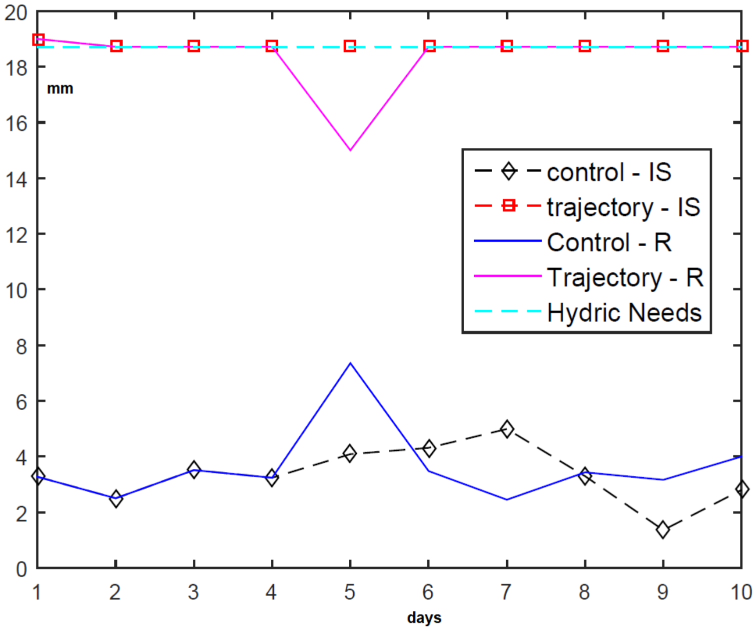

Figure 9: Replan base scenario with real moisture value on the fifth day is lower than expected [31]

Next, other similar example is shown to justify the use of the replan, and its qualitative validation. Let us now assume that the moisture sensor measures a different (higher) value than expected in the fifth day:

x_s = [19 18.7 18.7 18.7 22 18.7 18.7 18.7 18.7 18.7] (mm)

Results can be seen in Figure 10. As expected, the irrigation in the fifth day decrease. In this scenario, the amount of water used by the irrigation systems was 29.9 mm.

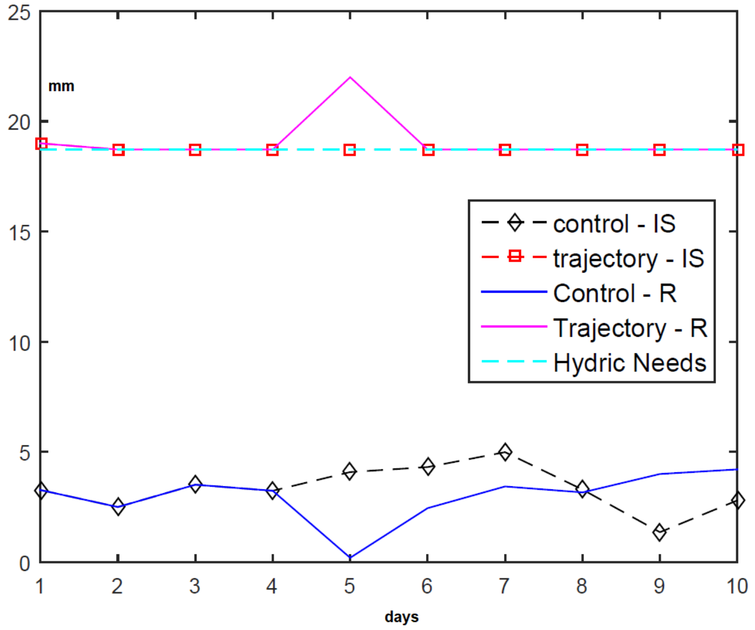

Figure 10: Replan base scenario with real moisture value on the fifth day is higher than expected [31].

From the above scenarios and its results, it is possible to conclude that the replan strategy is an important tool in the design of smart irrigation systems, being able to detect deviations from an ongoing prediction and acting accordingly. It allows to save water and keeps the crops safe.

Multiple irrigation points plan algorithm validation

Consider the following example. Assuming that are 4 irrigation points as in Figure 2.The installed capacity for each irrigation point is 24 mm, and, the irrigation plan for a six day period is for each irrigation point given by:

P[1; 1:6] = [10 15 15 11 5 8] mm
P[2; 1:6] = [20 11 20 11 6 5] mm
P[3; 1:6] = [20 21 15 12 7 16] mm
P[4; 1:6] = [12 24 24 22 9 5] mm.

As one may easily see, every irrigation point has the capacity to fulfil the needs. The irrigation performed (u) on every irrigation point in all the six days was as follows:

u[1; 1:6] = [10 15 15 11 5 8] mm
u[2; 1:6] = [20 11 20 11 6 5] mm
u[3; 1:6] = [20 21 15 12 7 16] mm
u[4; 1:6] = [12 24 24 22 9 5]mm.

Let us now consider the following situation. If all the inputs of the previous example are maintained, except the irrigation plan P[2; 1:6] = [20 11 34 11 6 5] mm, in day 3, the irrigation point is no longer able to fulfil the needs there.

In this situation, some water has to be taken from other irrigation points. On the other hand, it is possible to see that the installed capacity of all irrigation points is 96 mm. On the third day of this irrigation plan, the total water in need is 88 mm. This means the other three irrigation points are able to cope with the sudden needs in irrigation point number 2. In this example, the irrigation performed (u) on every irrigation point in all the six days is as follows:

u[1; 1:6] = [10 15 24 11 5 8] mm
u[2; 1:6] = [20 11 24 11 6 5] mm
u[3; 1:6] = [20 21 16 12 7 16] mm
u[4; 1:6] = [12 24 24 22 9 5] mm.

On the third day irrigation point number 1 used 24mm instead of 15mm, providing 9 mm to irrigation point number 2. Irrigation point number 3 used 16mm instead of 15mm, providing the extra 1mm to irrigation point number 2.

Finally, let us now consider the case where all the inputs remain as in the previous example and the irrigation plan P[2; 1:6] = [20 11 50 11 6 5] mm. On day 3, irrigation point number 2 has a need of 50mm. Its capacity is 24mm. Therefore, on that day the needs at all irrigation points are 104 mm, and the installed capacity for all 4 irrigation points is 96 mm. This means it is physically impossible to irrigate as planned. As expected, all the available water is used in day 3,

u[1; 1:6] = [10 15 24 11 5 8] mm
u[2; 1:6] = [20 11 24 11 6 5] mm
u[3; 1:6] = [20 21 24 12 7 16] mm
u[4; 1:6] = [12 24 24 22 9 5] mm.

but on day 3, there was water deficit (wd) and a message to the farmer is sent alerting for that fact at irrigation point number 2,

$$wd[1; 1::6] = [0\ 0\ 0\ 0\ 0\ 0\ ]\ \text{mm}$$
$$wd[2; 1::6] = [0\ 0\ 8\ 0\ 0\ 0\ ]\ \text{mm}$$
$$wd[3; 1::6] = [0\ 0\ 0\ 0\ 0\ 0\ ]\ \text{mm}$$
$$wd[4; 1::6] = [0\ 0\ 0\ 0\ 0\ 0\ ]\ \text{mm}.$$

CONCLUSIONS AND FUTURE WORK

This article proposed to design an improved mathematical model from the one presented in [21], that allows to have a smart irrigation system, provided that the irrigation requirements are feasible for a given data. Note that in [21] and [22] the solution for a similar problem was numerically validated, taking into account theoretical developments in Optimal Control, namely the Maximum Principle [22]. This new model incorporates new features like the slope of the soil, the possibility to include a percentage of water losses due to runoff, and a percentage of water losses if the soil is at field capacity. This improved model is able to address these features and a qualitative study was done and presented in the paper.

If the moisture sensors detect a value much different from the expected by the irrigation plan, a replan strategy is applied, in order to obtain a new irrigation plan. The proposed model incorporates a new constraint to impose that the soil moisture does not violate the state constraint, which forces to add to the control variable the value needed. Several examples were given that qualitatively validate the mathematical models proposed in this manuscript.

This is quite important, since it is not uncommon that weather forecasts are wrong, and therefore, the forecasted value of the moisture in the soil is wrong. Therefore, by introducing replan, it is possible to save water, and at the same time guarantee the safety of the crop.

A new approach to deal with multiple irrigation points is also proposed. Despite its simplicity, it allows to redistribute the available water in the case an irrigation point is not able to provide all that is needed accordingly to its irrigation plan. Once again, in draught periods, this is an important improvement, since it is possible that an irrigation point is not able to fulfil its duty, and neighbouring irrigation points might be able to help.

From the results shown in this manuscript, the authors believe that they have developed a valuable tool to use in a farm field and hopefully help to obtain a better and sustainable agriculture. In the future, other tests will be done, namely on site, in order calibrate the model and to obtain a reliable smart irrigation software prototype. Additional moisture sensors, and local weather data are to be acquired and considered as inputs to the mathematical model presented in this paper.

The research presented here, is integrated in a research project, funded by several national and international entities mentioned in the Funding Statement. It aims to design and create a smart irrigation prototype to be installed on a farm field in the North of Portugal.

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APPENDIX 1 –REPLAN ALGORITHM

```
IF1 $|x_i - x_s| > TOL$
   IF2 $x_s \geq x_i$
new_data = READ NEW WEATHER DATA;
     $x_i = x_s$
[$x, u$]=Irrigation_plan(new data, $x_i$)
ELSE
        IF3 $x_s \geq x_{min}$
new_data = READ NEW WEATHER DATA;
     $x_i = x_s$
     [$x, u$]=Irrigation_plan(new data, $x_i$)
          ELSE
new_data = READ NEW WEATHER DATA;
     $x_i = x_{min}$
$u_i$calculated such that $x_i = x_{min}$
              [$x, u$]=Irrigation_plan(new data, $x_i$)
        END3
    END2
END1
```

APPENDIX 2 –MULTIPLE IRRIGATION POINTS PLAN ALGORITHM

```
READ N – number of irrigation points
READ ND – number of days for the irrigation plan
READ Data(1..N) for each irrigation point  (weather, soil and crop)
READ irrigation capacity C(1..N) for each of the irrigation points
Cap_per_day=$\sum_{i=1}^{N} C(i)$
FOR i=1 TO N
P(i, 1..ND)=Irrigation plan(Data(i))
END
FOR day=1 TO ND
    FOR j=1 TO N
        IF P(j,day)<= C(j)
    u(j,day)=P(j,day); (irrigation for irrigation point j at day day)
      ELSE IF Needs_of_water(day)<=Cap_per_day
           [u(j,day),wd(j,day)]= Redistribute_water(P(j,day),Cap_per_day)
    ELSE
             Alert_farmer_for_water_needs(wd)
END
        END
    END
END
```