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Original Research Article

A Fuzzy Mixed-Integer Linear Programming Model for Optimizing Enhanced Weathering Using Red Mud

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ABSTRACT

There is an urgent need to limit average temperature rise to well below 2 °C by the end of this century. Negative emission technologies such as enhanced weathering are essential for large-scale CO_2 sequestration. Enhanced weathering accelerates the natural weathering of silicate minerals that results in the ultimate sequestration of carbon as dissolved bicarbonate ions in water. Silicate-rich non-hazardous industrial wastes, such as red mud waste from aluminum production, can also be used in enhanced weathering within a circular economy framework. There will be an ample supply of red mud for enhanced weathering because of the growing demand for aluminum in the global energy transition. However, there is limited literature on optimizing logistics of enhanced weathering operations. To address this research gap, this study develops a fuzzy mixed-integer linear programming model for red mud-based enhanced weathering networks, considering material sourcing, emissions, and costs. The model is demonstrated on a case study on Shandong Province, China, where 0.618 Mt CO_2/y of sequestration is achieved at a cost of 455 CNY/t CO_2 using just 44 % of the available red mud. Sensitivity analysis on cost and emission factors highlights the importance of renewable energy and carbon markets in carbon dioxide removal implementation. The findings highlight the potential use of non-hazardous industrial wastes for climate mitigation in the context of circular economy strategies.

KEYWORDS

Carbon dioxide removal, Decarbonization, Circular economy, Multi-objective optimization, Supply chain optimization

INTRODUCTION

Limiting global temperature rise to "well below 2 °C" requires drastic greenhouse gas (GHG) emissions reductions leading to carbon neutrality by mid-century [1]. With current GHG emissions trends, carbon dioxide removal (CDR) techniques, also known as negative emissions technologies (NETs), are almost certainly going to be needed at the gigaton (Gt) scale. Different NETs rely on various physical, chemical, and biological processes to transfer carbon from the atmosphere into other stocks or reservoirs [2]. For example, afforestation uses photosynthesis to store carbon in vegetation, while direct air capture uses engineered chemical processes to strip CO_2 from the atmosphere and store it under pressure in geological reservoirs. The extent of climate change mitigation benefits depends on the permanence, or durability, of the carbon storage mechanism [3].

Enhanced weathering (EW) stands out as a promising NET because it can deliver durable CDR while relying mostly on mature component processes [4]. EW takes advantage of the natural weathering reaction of alkaline minerals such as silicates with CO_2 and water to

produce bicarbonates which are then carried by the hydrological cycle into the oceans for long-term storage [5], meeting carbon credit eligibility requirements on permanence or durability. The minerals (e.g. silicate rocks like basalt, dunite, and laterite) are mined, pulverized to increase surface area, transported, and then applied to large areas of land [6]. All the component technologies needed for EW systems are commercially mature except for the techniques for measuring the exact amount of carbon sequestered [7]. GHG emissions from processes in the EW chain can negate some of the CDR benefits, particularly in locations where the electricity mix is highly dependent on fossil fuels [8]. Large-scale deployment of EW can also result in a broad range of environmental and socio-economic impacts [9]. Conversely, using EW on farmland can lead to substantial co-benefits due to enhanced soil fertility and reduced synthetic fertilizer carbon footprint [10]. Furthermore, registries have recognized that EW can be reconciled with standardized quantification and reporting requirements [11]. Non-hazardous industrial wastes (NHIW) such as metallurgical slag, cement kiln dust, red mud, waste concrete, and combustion ash can replace natural silicates in EW to eliminate the need for mining [12]. Since the standard disposal pathways for most NHIW are intended to isolate them from the environment, their use in EW generates clear additional CDR. EW has the potential to be a nexus solution for both carbon drawdown and circular economy ambitions [13].For example, the rising aluminum demand for the global energy transition will result in higher red mud waste generation that can be integrated into future EW networks [14]. There are already numerous commercial start-ups that produce CDR credits using EW [4]. However, ramping up EW from pilot scale demonstrations to Gt-level CDR will entail major logistical challenges [15].

Network optimization models have been proposed for planning EW systems in anticipation of future logistical challenges [16]. Model variants have also been developed for EW with single [17] and multiple [18] NHIW. However, past optimization models do not fully account for uncertainties in EW systems [19]. In particular, application rates can be limited by the decision-maker's risk tolerance to the accumulation of EW residues at the site. Fuzzy optimization was developed to incorporate such subjectivity in mathematical programming [20]. Conventional EW network models assume unrealistic, sharply-defined application limits at EW sites [16]. To address this research gap, this study develops a fuzzy mixed-integer linear programming (FMILP) model to optimize the allocation of NHIW sources to EW application sites in the context of voluntary carbon markets (VCM). The model maximizes CDR while balancing fuzzy constraints on application rate limits, CO_2 sequestration potential, and deterministic costs. The rest of the paper is organized as follows. Sections 2 and 3 give the formal problem statement and the model formulation. Section 4 demonstrates the model using a red mud case study. Section 5 states the conclusions and discusses future research opportunities.

PROBLEM STATEMENT AND CONCEPTUAL FRAMEWORK

The large–scale deployment of EW systems in VCMs will require systematic solutions to the logistical challenges of delivering available silicate sources to application sites to realize maximum CDR benefits [19]. This gap can be addressed using EW network optimization models. Only a finite amount of silicate materials is available at any given time from a given region; for example, red mud supply depends on the rate of processing of bauxite into alumina. In the assumed baseline, red mud is stockpiled at the point of generation where negligible geochemical weathering occurs; hence, the use of this waste in EW systems optimized for CDR guarantees additivity. In addition, the effectiveness of application is dose dependent and is limited by the risk of introducing contaminants to the site [21]. These factors impose a range of plausible values on the application rate. GHG emissions are generated from transporting and grinding the material [22], while CDR potential depends on NHIW's characteristics (i.e., chemical composition and particle size distribution) as well as

site conditions [23]. There are also accompanying costs to perform these activities [22]. CDR unit cost ultimately determines the viability of EW. Initial studies have shown that increasing CDR potential can result in higher costs, making the technology less competitive [22]. The deployment of the technology will thus require examining the trade-off between these objectives and identifying the optimal compromise solution, which can be addressed through fuzzy optimization [20].

The formal problem statement of the formulated model has the following assumptions and specifications:

- The EW system consists of m sources and n application sinks. Each i^{th} source (i = 1, 2, 3, … m) has a known and fixed annual production capacity and operates over a defined period.
- Each j^{th} application site (j = 1, 2, 3, … n) is characterized by fuzzy annual application rate limits (D_j^L to D_j^U) and a fixed cumulative rate limit (C_j).
- The total carbon footprint (f) and cost $(cost)$ associated with the system are treated as fuzzy parameters with linear degree of membership functions. The maximum potentially capturable CO_2 from the entire system, which results in the lowest carbon footprint, corresponds to the highest degree of satisfaction (λ) for the carbon footprint objective while a cost of zero corresponds to the highest degree of satisfaction for the cost objective. Note that a cost of zero indicates that the EW system was not implemented.

The problem is to determine the optimal allocation of NHIW from the sources to EW application sites (r_{ij}) to simultaneously maximize CDR considering the uncertainties in application rate limits of sites (A_j), maximize CO_2 sequestration potential, and minimize operating costs. The model assumes that all operations across the network begin simultaneously, with a constant allocation of NHIW per year throughout the system's operating life. The superstructure of the EW network resembles a supply chain network [17]. The network is illustrated in Figure 1 while the process flow is shown in Figure 2.

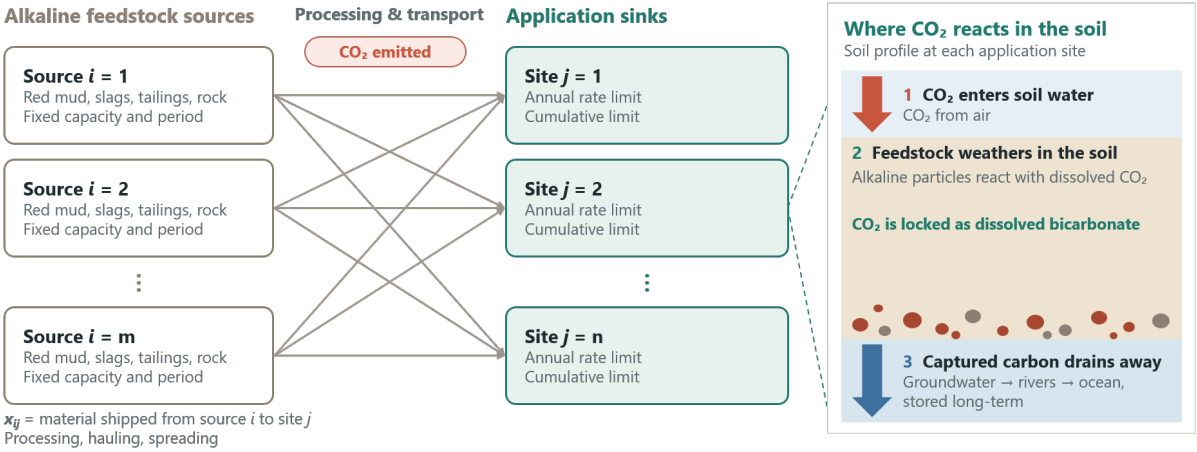


Figure 1: Enhanced weathering network[1]

[1] Schematic drafted with the assistance of Claude (Anthropic), an AI assistant, in Microsoft PowerPoint, then reviewed and edited by the authors. The figure contains no AI-generated imagery and represents no data

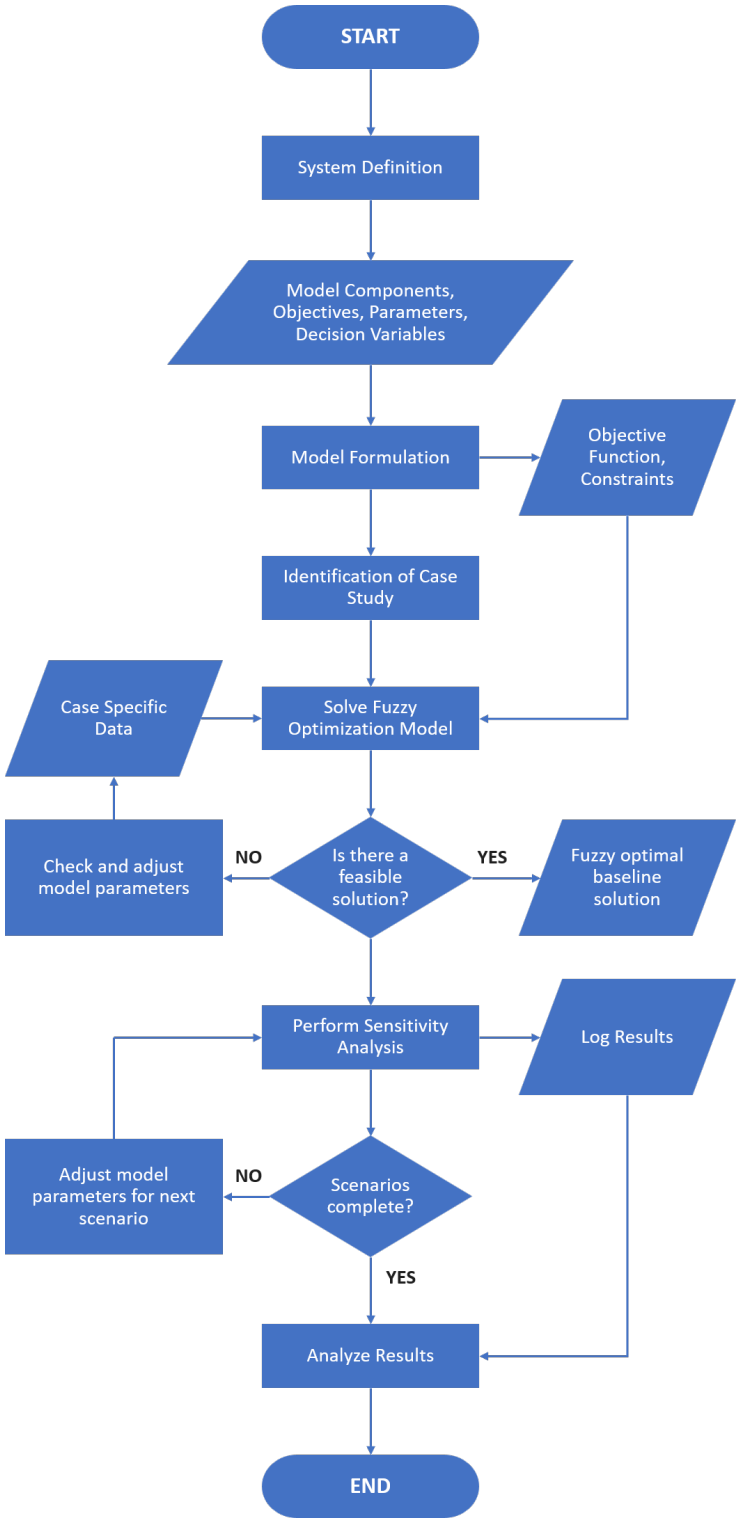


Figure 2: Process flow

OPTIMIZATION MODEL

The optimization model is formulated as a FMILP. The optimum of a fuzzy optimization problem is defined as the intersection of all its fuzzy goals and fuzzy constraints [20]. Fuzzy constraints relax the unrealistic assumption that feasible and infeasible solutions are separated by sharply-defined boundaries by introducing a gray area that allows a gradual transition in feasibility. This feature is important in the context of EW systems where site application rates

that determine the total CDR potential are themselves limited by the subjective tolerance of the decision-maker to the risk of accumulation of EW residues in soil [21]. Unlike frequency-based uncertainties that are best modelled via stochastic programming, this form of uncertainty is appropriately captured by fuzzy membership functions [24]. The degree to which each fuzzy goal or constraint is satisfied ranges from 0 to 1, with 0 indicating non-satisfaction, fractional values indicating partial satisfaction, and 1 indicating full satisfaction. Membership functions describe how the degrees of satisfaction respond to system variables. Zimmermann [24] proposed a fuzzy linear programming (FLP) formulation for problems where all the fuzzy sets have piecewise linear membership functions and where their set intersection is found via max-min aggregation. This approach is described as "symmetric" because fuzzy goals and constraints are mathematically indistinguishable. That formulation readily extends to FMILP through the addition of integer variables.

A base formulation was derived from previous models that considered the optimization of EW source-sink networks [16], that have also accounted for system uncertainties [25], and multiple objectives [17]. The model aims to maximize the aggregate degree of membership using Eq(1) and is formulated to ensure a Pareto optimal solution [26]. The optimal solution is subject to material flow constraints that include the balance of flows from the source (Eq(2)), indicating that the total amount of NHIW material delivered to any sink j from any source i cannot exceed what is available (S_i). A binary variable, b_i, is activated if NHIW from source i is transported to any sink j. The total amount of NHIW that can be applied in any given sink j from all sources i cannot exceed the allowable annual application rate, D_j, as indicated in (Eq(3)). In addition, the cumulative inflow to sink or application site j cannot exceed the maximum accumulation flow, A_j, as indicated in (Eq(4)), where T_i indicates the operating life of source i. Eq(5) indicates that material flows cannot be negative.

The carbon footprint, f, of the system is calculated using Eq(6). It consists of three terms that affect the system's carbon emissions. The first term refers to the sequestration achieved from enhanced weathering where α corresponds to the carbon sequestration potential per t of NHIW. This factor is based on the theoretical stoichiometric limit, corrected for the expected degree of reaction completion for the site; any inadvertent weathering of the NHIW in the original disposal path can also be deducted so that only the additional CDR is credited. A negative symbol precedes it since sequestration contributes towards reducing carbon emissions. The second term accounts for the emissions generated when the NHIW is transported from source to sink where Z is the carbon emission factor of the transport vehicle used, and X_{ij} is the distance between source i and sink j. The third term accounts for emissions generated from the comminution process where W is the energy consumed per t of NHIW processed, and γ_f is the carbon emission generated per unit of energy used. The magnitude of these 3 terms will depend on the total NHIW (r_{ij}) used throughout the operating lifetimes of the sources (T_i). The total system cost, $cost$, is calculated using Eq(7) and is comprised of the fixed (FC_i) and variable $(\kappa_{comminution})$ costs associated with comminution, transport costs $(\kappa_{Transport})$, and revenues generated from sequestered carbon, f, based on the carbon credit price (P).

Eq(8) defines the fuzzy membership function for application rates (A) where D_j^L and D_j^U are the lower and upper fuzzy limits for red mud application to any sink j and λ_{jA} is the degree of satisfaction achieved for sink j. Eq(9) defines the fuzzy membership function for the system's carbon footprint (B) where CF^L and CF^U are the lower and upper fuzzy limits for carbon footprint and λ_B is the degree of satisfaction achieved for the carbon footprint objective. Eq(10) defines the fuzzy membership function for cost (C) where TC^L and TC^U are the fuzzy limits for cost and λ_C is the degree of satisfaction achieved for the cost objective. Note that all fuzzy limits are exogenously defined. Additionally, the model applies max-min aggregation such that the over-all degree of satisfaction is maximized and should be less than

the individual degrees of satisfaction as indicated in Eq(11) to Eq(13). Eq. 14 defines the binary variable b_i which is activated once NHIW from source i becomes available ($b_i = 1$) for application to corresponding sinks as indicated in Eq(2).

Objective Function:

$$\lambda + \frac{1}{M}\left(\sum_{k} \quad \lambda_k\right) \tag{1}$$

Material Flow Constraints:

$$\sum_j r_{ij} \le b_i S_i \qquad \forall \ i \tag{2}$$

$$\sum_i r_{ij} \le D_j \qquad \forall \ j \tag{3}$$

$$\sum_i T_i r_{ij} \le A_j \qquad \forall \ j \tag{4}$$

$$r_{ij} \ge 0 \qquad \forall \ i,j \tag{5}$$

Carbon Footprint Constraints:

$$f = \sum_i \sum_j \left(-\alpha + ZX_{ij} + W\gamma_f\right) T_i r_{ij} \tag{6}$$

Cost Constraints:

$$cost = \sum_i b_i FC_i + \kappa_{Comminution}\left(\sum_i \sum_j r_{ij}\right) + \kappa_{Transport}\left(\sum_i \sum_j X_{ij} r_{ij}\right) - P(f) \tag{7}$$

Fuzzy Membership Constraints:

$$d_j \le D_j^U - \lambda_{jA}\left(D_j^U - D_j^L\right) \qquad \forall \ j \tag{8}$$

$$f \le CF^U - \lambda_B (CF^U - CF^L) \tag{9}$$

$$cost \le TC^U - \lambda_C (TC^U - TC^L) \tag{10}$$

$$\lambda \le \lambda_{jA} \qquad \forall \ j \tag{11}$$

$$\lambda \le \lambda_B \tag{12}$$

$$\lambda \le \lambda_C \tag{13}$$

$$b_i \in \{0,1\} \qquad \forall_i \tag{14}$$

CASE STUDY

A case study is formulated to assess EW using red mud as the NHIW of interest in Shandong Province, China, based on 17 sources (Table 1) and 4 coastal sink sites (Table 2) based on the work of Jia et al. [27]. The operations are assumed to occur in the context of a VCM. The distances between sources and sinks are summarized in Table 3. The model is implemented using the optimization software LINGO 20.0 performed on a MacBook Pro 2022 (Apple M2, 8 GB RAM) running on macOS. Global optima were achieved within an

average solution time of 7s. Since the formulated model is MILP, checking for alternative solutions can be done using integer cut constraints which disallow previously identified solutions. This procedure can also identify near-optimal solutions. This process can be done automatically using LINGO by activating the k-best solution feature to activate integer-cuts. The latter approach was used here.

Table 1: Red mud production rate of sources (S_i)

	Source	Production Rate (kt/y)
S1	Jinan	226.00
S2	Qingdao	97.00
S3	Weihai	74.70
S4	Taian	1,028.60
S5	Yantai	1,242.80
S6	Linyu	434.20
S7	Rizhao	89.70
S8	Dongying	60.90
S9	Zibo	1682.20
S10	Weifang	171.40
S11	Jining	144.50
S12	Zaozhuang	53.40
S13	Binzhou	25,777.00
S14	Dezhou	661.40
S15	Liaocheng	4,273.50
S16	Heze	158.90
S17	Laiwu	190.80
	Total	**36,367.70**

Table 2: Application rate limit (D_j) and maximum accumulation in sinks (A_j)

Sink (j)		Application Rate (kt/y)		Maximum Accumulation (kt) in 5 years
		Lower Limit, D_j^L	Upper Limit, D_j^U	
D1	Lijin	73.45	18,434.93	553,047.97
D2	Hekou	447.77	112,390.08	3,371,702.49
D3	Zhanhua	67.04	16,825.88	504,776.37
D4	Changyi	88.35	22,175.18	665,255.31

Table 3. Distance between sources and sinks (in km)

	D1	D2	D3	D4
S1	159.22	213.25	165.49	226.42
S2	294.77	317.77	334.32	160.01
S3	419.45	440.65	460.44	288.55
S4	228.34	286.7	236.71	265.92
S5	349.73	372.23	387.84	216.6
S6	316.1	341.92	341.92	341.92
S7	299.44	322.7	332.57	188.32
S8	40.73	59.43	74.71	116.67
S9	84.65	140.69	115.07	134.45
S10	139.74	161.5	180.13	34.14
S11	347.52	391.53	340.26	381.22
S12	356.4	406.45	371.08	344.34
S13	29.99	87.22	42.12	163.14
S14	197.4	216.99	171.69	321.7
S15	250.53	308.85	257.3	332.9
S16	394.43	454.46	402.31	466.09
S17	175.53	225.62	188.99	215.67

Red mud production rates are assumed constant over a 5-year period following Jia et al. [27]. Stoichiometric analysis based on Zia-Ur-Rehman et al. [28] determines application limits, factoring in mineral composition, available area [27], rainfall [29], and soil conditions [30]. Carbon sequestration potential (α) is set at 128 kg CO_2/t [12] and balanced against emissions from transportation and comminution. Transportation emissions are based on source-to-sink distances [27] and an emission factor (Z) of 62 kg CO_2/km t using road heavy goods vehicle [31] with emissions data from CEFIC [32]. Comminution emissions are calculated using Bond's law, assuming a work index of 20 kWh/t [31] and particle size reduction from 56.88 μm to 10 μm [33], resulting in the energy consumption of 36.73 kWh/t red mud with energy emissions quantified using a grid factor of $\gamma_f = 1.004$ kg CO_2/kWh for Shandong [34]. System costs include a transportation cost factor of $\kappa_{Transport} = 0.3$ CNY/km/t, which assumes a 3-axis rigid truck with 30 t payload with an average diesel fuel cost of 12.68 CNY/l [27]. It also includes a fixed equipment cost of FC =51.1 million CNY, and comminution cost factor $\kappa_{Comminution} = 3.06$ CNY/t, assuming the use of grinding equipment (ball mill media) to achieve the desired size reduction target of 10μm [31].

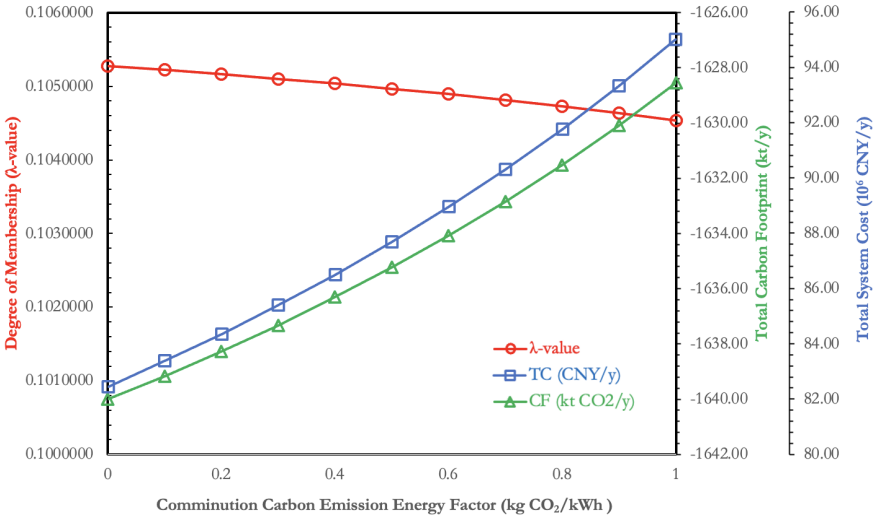
The fuzzy limits for total carbon footprint and system cost are calculated to be at a range of (–15.6, 0) Mt CO_2 and (–15,268.8, 1,880.9) million CNY, respectively. Negative value for system cost indicates profitable EW system. The fuzzy lower and upper limits for cost were

obtained by minimizing the carbon footprint (Eq(6)) subject to Eq(2) to Eq(5), Eq(7), and Eq(14), using a carbon credit cost of USD 150/t CO_2 and USD 0/t CO_2 respectively, where a conversion factor of USD 1 = 7.25 CNY was used. The fuzzy upper limit for total carbon footprint ($CF^U = 0\ \mathrm{Mt\ CO_2}$) corresponds to the footprint if EW was not implemented while the lower limit ($CF^L = -15.6\ \mathrm{Mt\ CO_2}$) was the lowest achievable footprint using a carbon credit cost of USD 150/t CO_2. The application rate should be within the lower and upper fuzzy limits which were based on user-defined, site-specific dosage levels for EW. The choice of values should consider epistemic uncertainties in the weathering side reactions as well as the risk tolerance of the decision maker [35].

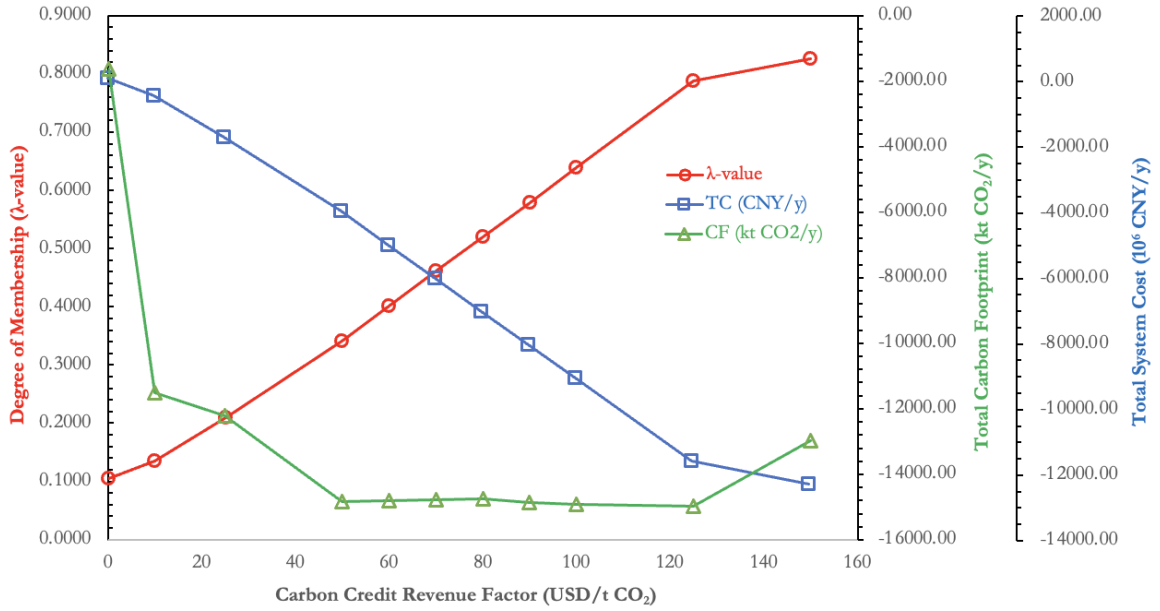
The gathered limits are then used as model parameters in the main model, and the optimal solution is determined by solving Eq. (1) subject to constraints indicated in Eq. (2) – Eq. (14) while using a carbon credit cost of USD 0/t CO_2.

The optimal solution results in λ = 0.1045, signifying that every fuzzy goal or constraint is at least 10.45% satisfied. This corresponds to a carbon sequestration of 1.63 Mt/y and a total system cost of 95.1 million CNY, translating to 58.39 CNY/t CO_2, which is lower than the range estimated by Renforth [31] having a lower limit of 507.5 CNY/t CO_2. The shortfall may be due to the higher costs of extracting and crushing freshly quarried rocks [31]. While the membership degree is below average, this is balanced by significant cost savings and can be viewed as a trade-off between higher costs for better environmental outcomes versus more cost-effective solutions with slightly reduced environmental benefits. The resulting network connects one source, Binzhou, to the sink site, Lijin, corresponding to an annual red mud allocation of 3,648.57 kt/y. The optimized network flow consumes 10.03% of the annual total red mud waste.

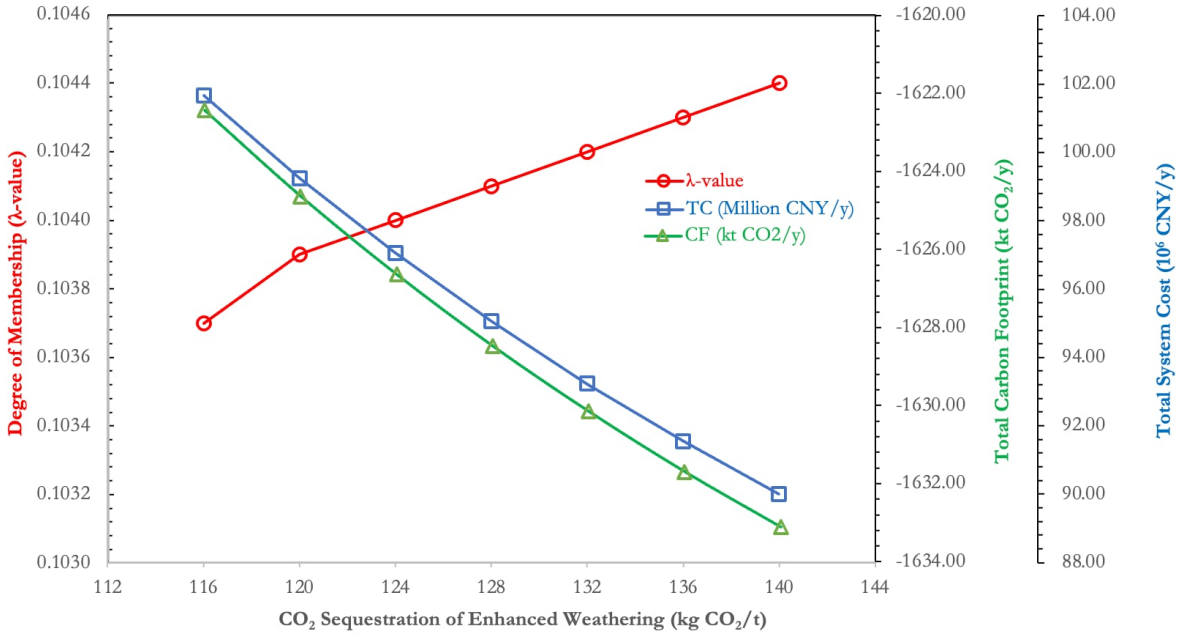
The optimized results demonstrate the potential of using NHIWs like red mud for EW as a NETs as indicated by the resulting negative carbon footprint for an adequately priced carbon market. Though the resulting baseline optimal solution has a positive total system cost, this can be offset by increasing the carbon credit value of the sequestered carbon. This scheme supports the integration of EW into existing waste management policies, showing how red mud can be repurposed to support a circular economy.



(a)



(b)



(c)

Figure 3: Optimal Solution for Scenarios Varying (a) Carbon Emission Energy and (b) Carbon Credit Price

Sensitivity analysis was then conducted on 3 parameters – 1) carbon emissions from energy required in the comminution, 2) carbon credit price, and 3) CO_2 sequestration potential of red mud. Results are illustrated in Figure 3 while a summary of the conditions used for the scenarios illustrated in Figure 3 is found in Table 4.

Table 4. Sensitivity Analysis scenarios conducted

Scenario	Comminution Carbon emission energy factor (γ_f, in kg CO_2/kWh)	Carbon Credit price (P, in USD/t CO_2)	Carbon sequestration potential $\left(\alpha, \text{kg}\frac{CO_2}{\text{t}}\text{ red mud}\right)$
Baseline	1.004	0	128
Scenario A	0–1.004	0	128
Scenario B	1.004	0–150	128
Scenario C	1.004	0	116–140

As carbon emissions from power consumption in comminution processes vary by region, the efficiency of EW systems will be affected. Higher factors in fossil fuel-dependent areas like China will have decreased sequestration efficiency compared to countries with more renewables in the energy mix [36]. This highlights the need to consider energy sources when optimizing EW systems.

For the sensitivity analysis on carbon credit price, the graph reveals the trend of the total system cost to have an inverse relationship with the carbon credit price indicating revenues with the increase in carbon credit price. The fuzzy optimization model forces to reduce the carbon footprint of the system from the baseline to achieve a compromising solution between the identified objectives. Results show that revenues can be generated even at a low carbon credit price of USD 4.84/t CO_2 suggesting that high carbon credit price can further drive carbon reduction efforts. However, it is important to note that further increasing carbon credit price does not necessarily result in further decreasing the carbon footprint as revenues result in reduced carbon sequestration requirements as seen for a price of USD 150/t CO_2.

For variations in the carbon sequestration potential of EW using red mud, results suggest that increasing the sequestration potential results in a decrease in total carbon footprint and cost of the system with a steeper decline in the system's carbon footprint.

The sensitivity analyses provide insights into how the expansion of the carbon market and improvement of the carbon credit system can significantly help the implementation of NETs, such as EW. It also displays possible critical developments for the CDR industry with the developing valuation of carbon credits to not only sustain efforts on sequestering carbon emissions but also earn to make significant profit.

CONCLUSION

The study has developed an FMILP model for an EW network using NHIW such as red mud. The model considers critical factors like material supply and application rate limits, as well as process emissions and cost constraints. The most important feature of this model is the use of fuzzy membership functions to capture the decision-maker's subjective risk tolerance to the potential accumulation of EW residues at the application sites. It is flexible enough to be easily modified for EW with a wide range of silicate-rich industrial wastes. Findings from the formulated red mud case study highlight the feasibility of integrating EW strategies into climate action plans while also promoting the principles of circular economy if carbon credits generated from the sequestered carbon offsets costs associated with EW infrastructure. Results of the case study indicate a minimal carbon credit price of USD 4.84/t CO_2 to achieve substantial carbon footprint reductions alongside revenues. In addition, increasing carbon credit price does not necessarily result in further carbon footprint reductions. Models based on this formulation can facilitate the ramp up of EW from current demonstration levels to future Gt-scale commercial operations. These models reveal the

trade-offs between maximizing sequestration and minimizing costs that can help policymakers design targeted subsidies or financial incentives through carbon credits. These can also guide land-use regulations by identifying ideal application sites and support the integration of EW strategies into industrial and waste management sectors. In future work, this study can be extended by considering alternative types of EW sites (e.g. terrestrial, marine), optimizing logistics through transshipment hubs, and selecting alternative modes of transportation. In cases where some geochemical weathering already occurs in the baseline fate of the NHIW, numerical factors can easily be added to these models to account only for the additional CDR from EW operations. Novel network optimization models can also be field-tested by EW start-ups.

NOMENCLATURE

Subscripts and Superscripts

I	EW Sources
J	EW Sinks
K	Criterion

Model Parameters

A	Carbon sequestration potential	[kg CO_2/t NHIW]
γ_f	Carbon emission energy factor	[kg CO_2/kWh]
$\kappa_{Transport}$	Cost factor of transportation process	[CNY/km/t NHIW]
$\kappa_{Comminution}$	Cost factor of comminution process	[CNY/t NHIW]
A_j	Max EW material accumulation in sink j	[t NHIW]
CF^L	Lower limit of carbon footprint	[t CO_2/y]
CF^U	Upper limit of carbon footprint	[t CO_2/y]
D_j^L	Lower limit for EW application in sink j	[t NHIW/y]
D_j^U	Upper limit for EW application in sink j	[t NHIW/y]
FC_i	Fixed cost for comminution equipment in source i	[CNY]
M	Arbitrary large number	[dimensionless]
P	Carbon credit price	[USD/t CO_2]
S_i	Annual NHIW material production	[t NHIW/y]
T_i	Operating life of source i	[y]
TC^L	Lower limit for total cost	[CNY/y]
TC^U	Upper limit for total cost	[CNY/y]
W	Energy consumed in the comminution process	[kWh/t NHIW]
X_{ij}	Distance from source i to sink j	[km]
Z	Carbon emission factor of transportation mode and vehicle	[kg CO_2/km/t NHIW]

Decision Variables

λ	Over-all degree of membership	[dimensionless]
λ_k	Degree of membership for sink j in criterion k	[dimensionless]
b_i	Binary variable for activation of source i	[dimensionless]
Cost	Total cost	[CNY/y]
d_j	Annual EW material application	[t NHIW/y]
F	Carbon footprint	[t CO_2/y]
r_{ij}	Annual flowrate of crushed NHIW from source	[t NHIW/y]

i to sink j

Abbreviations

CDR	Carbon dioxide removal
EW	Enhanced weathering
FLP	Fuzzy linear programming
FMILP	Fuzzy mixed integer linear programming
GHG	Greenhouse gas
NET	Negative emissions technologies
NHIW	Non-hazardous industrial waste

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